Example: The kinship domain

- The objects in this domain are people.
- They have properties such as gender.
  - Unary predicates: Male and Female.
- They are related by relations such as parenthood, brotherhood, marriage, ...
  - Binary predicates: Parent, Sibling, brother, Sister, Child, Doughter, Son, Wife, Husband, Grandparent, Grandchild,
- Use functions for Mother and Father
- One's mother is one's female parent:

$$\forall m, c \ \text{Mother}(c) = m \Leftrightarrow \text{Femala}(m) \wedge \text{Parent}(m, c)$$

- One's husband is one's male spouse:

$$\forall w, h \ \text{Husband}(h, w) \Leftrightarrow \text{Male}(h) \wedge \text{Spouse}(h, w)$$

- Male and female are disjoint categories:

$$\forall x \ \text{Male}(x) \Leftrightarrow \neg\text{Female}(x)$$

- Parent and child are inverse relations:

$$\forall p, c \ \text{Parent}(p, c) \Leftrightarrow \text{Child}(c, p)$$

- A grandparent is a parent of one's parent:

$$\forall g, c \ \text{Grahdparent}(g, c) \Leftrightarrow \exists p \ \text{Parent}(g, p) \wedge \text{Parent}(p, c)$$

- A subling is another child of one's parent:

$$\forall x, y \ \text{Subling}(x, y) \Leftrightarrow x \neq y \wedge \exists p \ \text{Parent}(p, x) \wedge \text{Parent}(p, y)$$

Additional Inference Rules for FOL

- Substitution $\text{SUBST}(\theta, \alpha)$:
  - Applying substitution (or binding list) θ to sentence α.
  - Example:

    $\text{SUBST}(\{x/\mathit{Tom}, y/\mathit{Eva}\}, \mathit{Likes}(x, y)) \ = \ \mathit{Likes}(\mathit{Tom}, \mathit{Eva})$

- Additional inference rules (involving quantifiers) for FOL:
  - Universal elimination:

$$\frac{\forall v \ \alpha}{\text{SUBST}(\{v/g\}, \alpha)}$$

    where v is a variable and g is a ground term (i.e. a term containing no variables).
  - Existential elimination:

$$\frac{\exists v \ \alpha}{\text{SUBST}(\{v/k\}, \alpha)}$$

    where v is a variable and k is a constant not appearing elsewhere in the KB.
  - Existential introduction:

$$\frac{\alpha}{\exists v \ \text{SUBST}(\{g/v\}, \alpha)}$$

    where v is a variable that does not occur in α and g is a ground term that occurs in α.

UNIFICATION

- Intuitive idea:
  - Given two atomic sentences p and q, a substitution (if exists) that would make p and q look identical is returned.
- More formally:
  - $\textsc{Unify}(p,q) = \theta$ where $\textsc{Subst}(\theta,p) = \textsc{Subst}(\theta,q)$.
  - θ is called the *unifier* of p and q.
- Renaming of variables may be necessary to avoid name clashes between sentences.
- Example:
  Let the KB contains:
  - Knows(John, Jane)
  - Knows(y, Mary)
  - Knows(y, Mother(y))
  - Knows(x, Elizabeth)
- Unifying with Knows(john,x):
  - UNIFY(Knows(John,x), Knows(john,Jane))) = {x/Jane}
  - UNIFY(Knows(John,x), Knows(y, Mary)) = {x/Mary, y/John}
  - UNIFY(Knows(John,x), Knows(y, Mother(y))) = {y/John, x/Mother(John)}
  - UNIFY(Knows(John,x), Knows(x, Elizabeth)) = fail

CANONICAL FORMS FOR RESOLUTION

- Conjunctive normal form (CNF) - every sentence is a disjunction of literals.
- Implicative normal form (INF) - each sentence is an implication with a conjunction of atoms on the left and a disjunction of atoms on the right.
- Example
  Conjunctive normal form:
  - $\neg P(w) \vee Q(w)$
  - $P(x) \vee R(x)$
  - $\neg Q(y) \vee S(y)$
  - $\neg R(z) \vee S(z)$

  Implicative normal form:
  - $P(w) \Rightarrow Q(w)$
  - $Ture \Rightarrow P(x) \vee R(x)$
  - $Q(y) \Rightarrow S(y)$
  - $R(z) \Rightarrow S(z)$