Conversion to Normal form

1. Eliminate implications .
2. Move negations down to the atomic formulas
3. Purge existential quantifiers
4. Rename variables , if necessary
5. Move the universal quantifiers to the left
6. Move the disjunctions down to the literals
7. Eliminate the conjunctions
8. Rename variable , if necessary
9. Purge the universal quantifiers

Example: Converting Axioms to Normal Form

$$\begin{aligned}\forall x[\mathrm{Brick}(x) \Rightarrow (&\exists y[\mathrm{On}(x,y) \,\&\, \neg\mathrm{Pyramid}(y)]\\ &\&\ \neg\exists y[\mathrm{On}(x,y) \,\&\, \mathrm{On}(y,x)]\\ &\&\ \forall y[\neg\mathrm{Brick}(y) \Rightarrow \neg\mathrm{Equal}(x,y)])]\end{aligned}$$

- Eliminate implications.

substitute $\neg E_1 \vee E_2$ for $E_1 \Rightarrow E_2$

$$\begin{aligned}\forall x[\neg\mathrm{Brick}(x) \vee (&\exists y[\mathrm{On}(x,y) \,\&\, \neg\mathrm{Pyramid}(y)]\\ &\&\ \neg\exists y[\mathrm{On}(x,y) \,\&\, \mathrm{On}(y,x)]\\ &\&\ \forall y[\neg(\neg\mathrm{Brick}(y)) \vee \neg\mathrm{Equal}(x,y)])]\end{aligned}$$

- Move negations down to the atomic formulas.

Doing this requires a number of identities.

$$\begin{aligned}&\neg(E_1 \,\&\, E_2) \rightarrow (\neg E_1) \vee (\neg E_2)\\ &\neg(E_1 \vee E_2) \rightarrow (\neg E_1) \,\&\, (\neg E_2)\\ &\neg(\neg E_1) \rightarrow E_1\\ &\neg\forall x[E_1(x)] \rightarrow \exists x[\neg E_1(x)]\\ &\neg\exists x[E_1(x)] \rightarrow \forall x[\neg E_1(x)]\end{aligned}$$

$$\begin{aligned}\forall x[\neg\mathrm{Brick}(x) \vee (&\exists y[\mathrm{On}(x,y) \,\&\, \neg\mathrm{Pyramid}(y)]\\ &\&\ \forall y[\neg\mathrm{On}(x,y) \vee \neg\mathrm{On}(y,x)]\\ &\&\ \forall y[\mathrm{Brick}(y) \vee \neg\mathrm{Equal}(x,y)])]\end{aligned}$$

Example (CONT'D)

- Purge existential quantifiers.

$$\exists y[\mathrm{On}(x,y) \ \&\ \neg\mathrm{Pyramid}(y)]$$

Functions that eliminate the need for existential quantifiers are called *Skolem functions.*

$$\begin{aligned}\forall x[\neg\mathrm{Brick}(x) \vee ((\mathrm{On}(x,\mathrm{Support}(x)) \ \&\ \neg\mathrm{Pyramid}(\mathrm{Support}(x)))\\ \&\ \forall y[\neg\mathrm{On}(x,y) \vee \neg\mathrm{On}(y,x)]\\ \&\ \forall y[\mathrm{Brick}(y) \vee \neg\mathrm{Equal}(x,y)])]\end{aligned}$$

- Rename variables, as necessary, so that no two variables are the same.

$$\begin{aligned}\forall x[\neg\mathrm{Brick}(x) \vee ((\mathrm{On}(x,\mathrm{Support}(x)) \ \&\ \neg\mathrm{Pyramid}(\mathrm{Support}(x)))\\ \&\ \forall y[\neg\mathrm{On}(x,y) \vee \neg\mathrm{On}(y,x)]\\ \&\ \forall z[\mathrm{Brick}(z) \vee \neg\mathrm{Equal}(x,z)])]\end{aligned}$$

Example (CONT'D)

- Move the universal quantifiers to the left.

This works because by now each quantifier uses a unique variable name no confusion results from leftward movement.

$$\begin{aligned}\forall x\forall y\forall z[\neg\mathrm{Brick}(x) \vee ((\mathrm{On}(x,\mathrm{Support}(x)) \ \&\ \neg\mathrm{Pyramid}(\mathrm{Support}(x)))\\ \&\ \neg\mathrm{On}(x,y) \vee \neg\mathrm{On}(y,x)\\ \&\ \mathrm{Brick}(z) \vee \neg\mathrm{Equal}(x,z))]\end{aligned}$$

- Move the disjunctions down to the literals.

Since all this requires is to move the $\vee$s inside the &s, we need only use one of the distributive laws:

$$E_1 \vee (E_2 \ \&\ E_3) \Rightarrow (E_1 \vee E_2) \ \&\ (E_1 \vee E_3)$$

$$\begin{aligned}\forall x\forall y\forall z[(\neg\mathrm{Brick}(x) \vee (\mathrm{On}(x,\mathrm{Support}(x)) \ \&\ \neg\mathrm{Pyramid}(\mathrm{Support}(x))))\\ \&\ (\neg\mathrm{Brick}(x) \vee \neg\mathrm{On}(x,y) \vee \neg\mathrm{On}(y,x))\\ \&\ (\neg\mathrm{Brick}(x) \vee \mathrm{Brick}(z) \vee \neg\mathrm{Equal}(x,z))]\end{aligned}$$

$$\begin{aligned}\forall x\forall y\forall z[(\neg\mathrm{Brick}(x) \vee \mathrm{On}(x,\mathrm{Support}(x)))\\ \&\ (\neg\mathrm{Brick}(x) \vee \neg\mathrm{Pyramid}(\mathrm{Support}(x)))\\ \&\ (\neg\mathrm{Brick}(x) \vee \neg\mathrm{On}(x,y) \vee \neg\mathrm{On}(y,x))\\ \&\ (\neg\mathrm{Brick}(x) \vee \mathrm{Brick}(z) \vee \neg\mathrm{Equal}(x,z))]\end{aligned}$$

Example (CONT'D)

- Eliminate the conjunctions.

$$\forall x[\neg \mathrm{Brick}(x) \vee \mathrm{On}(x, \mathrm{Support}(x))]$$
$$\forall x[\neg \mathrm{Brick}(x) \vee \neg \mathrm{Pyramid}(\mathrm{Support}(x))]$$
$$\forall x \forall y[\neg \mathrm{Brick}(x) \vee \neg \mathrm{On}(x, y) \vee \neg \mathrm{On}(y, x)]$$
$$\forall x \forall z[\neg \mathrm{Brick}(x) \vee \mathrm{Brick}(z) \vee \neg \mathrm{Equal}(x, z)]$$

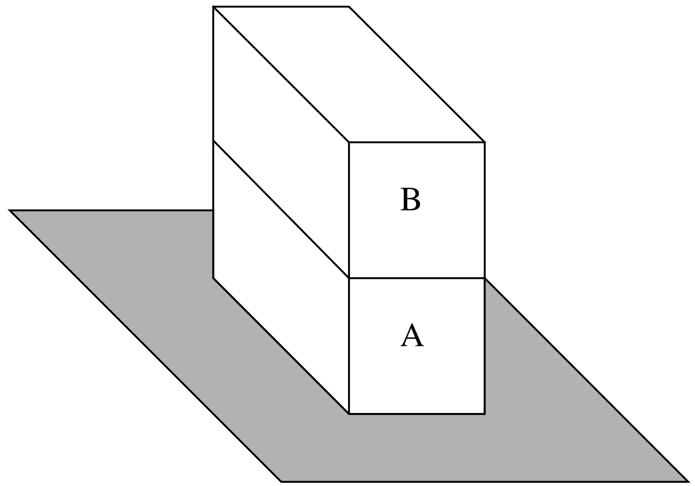
- Rename all the variables, as necessary, so that no two variables are the same.

$$\forall x[\neg \mathrm{Brick}(x) \vee \mathrm{On}(x, \mathrm{Support}(x))]$$
$$\forall w[\neg \mathrm{Brick}(w) \vee \neg \mathrm{Pyramid}(\mathrm{Support}(w))]$$
$$\forall u \forall y[\neg \mathrm{Brick}(u) \vee \neg \mathrm{On}(u, y) \vee \neg \mathrm{On}(y, u)]$$
$$\forall v \forall z[\neg \mathrm{Brick}(v) \vee \mathrm{Brick}(z) \vee \neg \mathrm{Equal}(v, z)]$$

- Purge the universal quantifiers.

$$\neg \mathrm{Brick}(x) \vee \mathrm{On}(x, \mathrm{Support}(x))$$
$$\neg \mathrm{Brick}(w) \vee \neg \mathrm{Pyramid}(\mathrm{Support}(w))$$
$$\neg \mathrm{Brick}(u) \vee \neg \mathrm{On}(u, y) \vee \neg \mathrm{On}(y, u)$$
$$\neg \mathrm{Brick}(v) \vee \mathrm{Brick}(z) \vee \neg \mathrm{Equal}(v, z)$$

An Example



On (B , A)

On (A, T able)

$\forall$ x $\forall$ y [On (x,y) $\Rightarrow$ Above (x,y)]

$\forall$ x $\forall$ y $\forall$ z [Above (x,y) Λ Above (y,z) $\Rightarrow$ Above (x,z)]

Prove:

Above (B , Table)

Convert to Conjunctive Normal Form:

$\text{On}(B, A)$ (1)

$\text{On}(A, \text{Table})$ (2)

$\neg \text{On}(u,v) \vee \text{Above}(u,v)$ (3)

$\neg \text{Above}(x,y) \vee \neg \text{Above}(y,z) \vee \text{Above}(x,z)$ (4)

Negate the theorem to be proved:

$\neg \text{Above}(B, \text{Table})$ (5)

Proof by resolution:

$\neg\text{Above}(B,y) \vee \neg\text{Above}(y, \text{Table}) \vee \text{Above}(B, \text{Table})$ (4)

$\neg \text{Above}(B, \text{Table})$ (5)

$\neg \text{Above}(B,y) \vee \neg \text{Above}(y, \text{Table})$ (6)

$\neg \text{On}(y, \text{Table}) \vee \text{Above}(y, \text{Table})$ (3)

$\neg \text{Above}(B,y) \vee \neg \text{Above}(y, \text{Table})$ (6)

$\neg \text{On}(y, \text{Table}) \vee \neg \text{Above}(B, y)$ (7)

$\neg \text{On}(B, y) \vee \text{Above}(B, y)$ (3)

$\neg \text{On}(y, \text{Table}) \vee \neg \text{Above}(B, y)$ (7)

$\neg \text{On}(B, y) \vee \neg \text{On}(y, \text{Table})$ (8)

$\text{On}(B, A)$ (1)

$\neg \text{On}(B, A) \vee \neg \text{On}(A, \text{Table})$ (8)

$\neg \text{On}(A, \text{Table})$ (9)

$\text{On}(A, \text{Table})$ (2)

$\neg \text{On}(A, \text{Table})$ (9)

Nil

Resolution Refutation

- Whoever can read is literate .
  $(\forall x)\ [R(x) \Rightarrow L(x)]$

- Dolphins are not literate .
  $(\forall x)\ [D(x) \Rightarrow \neg L(x)]$

- Some dolphins are intelligent.
  $(\exists x)\ [D(x)\ \Lambda\ I(x)]$

- The set of c lauses corresponding to the above statements:
  $\neg R(x)\ V\ L(x)$
  $\neg D(y)\ V\ \neg L(y)$
  $D(A)$
  $I(A)$

- Want to prove the statement: Some who are intelligent cannot read.
  $(\exists x)\ [I(x)\ \Lambda\ \neg R(x)]$

- The negation of the theorem to be proved:
  $\neg I(z)\ V\ R(z)$

Derivation Graph