Types of Reasoning

- DEDUCTION - The process of reasoning in which a conclusion follows necessarily from the stated premises; inference by reasoning from the general to the specific.
- INDUCTION - A principle of reasoning to a conclusion about all members of a class from examination of only a few member of the class; broadly, reasoning from the particular to the general.
- ABDUCTION - This is a form of deductive logic which provides only a "plausible inference."

Abduction

- Reasoning by abduction:
  - From: a ⇒ b and a
    Infer: a
  - Example:

$$\frac{\text{drunk(X)} \Rightarrow \neg\ \text{walk-straight(X)}, \quad \neg\ \text{walk-straight(John)}}{\text{drunk(John)}}$$

  - Abduction tells us that one hypothetical explanation for John's inability to walk straight is that he is drunk.
- Abduction is not a legal inference, it allows false conclusions:

$$\frac{\forall\ \text{x has-cancer(X)} \Rightarrow \text{feels-terrible(X)}, \quad \text{feels-terrible(Bill)}}{\text{has-cancer(Bill)}}$$

- Abduction is only plausible (probable) inference.

Uncertainty and Probability

- Pure logical agents cannot deal with uncertainties.
- *Uncertainties* are exceptions to rules.

$$\frac{\text{Lawyer(X)} \Rightarrow \text{Rich(X)}, \qquad \text{Lawyer(John)}}{\text{Rich(John)}}$$

A lawyer might not be rich because of

- lack of customers
- bankrupted business
- addiction to gambling
- frequent divorces
- ...

- Solution in logic:

  Lawyer(X) $\wedge$ $\neg$lackCustomers(X) $\wedge$ $\neg$ badBusiness(X)
  $\wedge$ $\neg$ gambler(X) $\wedge$ stableMarriage(X) $\wedge$ ... $\Rightarrow$ Rich(X)

- It is impossible to list all the possible exceptions.
- Even if we can, the rule becomes too complex to be useful.
- Probabilistic solution: Quantify the rule using probability

  Lawyer(X) $\Rightarrow$ Rich(X) (0.8)

Utility

- Probability alone is not enough for rational decision making.
- Example: The oil wildcatter problem (Raffia 1968):
  - Decision: Drill?
  - Probabilities of outcome if drill.

dry	wet	soaking
.500	.300	.200

  - Drill or not drill?
- We also need utilities:

dry	wet	soaking
-$100K	$100K	$500K

- Decision theory:
  - Combining probability theory with utility theory
  - Principle of maximum expected utility:
    * An agent is rational if and only if it chooses the action that yields the highest expected utility, averaged over all the possible outcomes of the action.
      1. If not drill, $\$0K$.
      2. If drill,

$$-100 \times 0.5 + 100 \times 0.3 + 500 \times 0.2 = \$80K$$

      3. Rational decision: drill.

Decision-Theoretic Agents

function DT-AGENT(*percept*) **returns** an *action*
  static: a set probabilistic beliefs about the state of the world

  calculate updated probabilities for current state based on
    available evidence including current percept and previous action
  calculate outcome probabilities for actions,
    given action descriptions and probabilities of current states
  select *action* with highest expected utility
    given probabilities of outcomes and utility information
  return *action*

Axioms for Probability

- All probabilities are between 0 and 1:

$$0 \leq P(A) \leq 1$$

- Necessarily true propositions have probability 1:

$$P(True) = 1$$

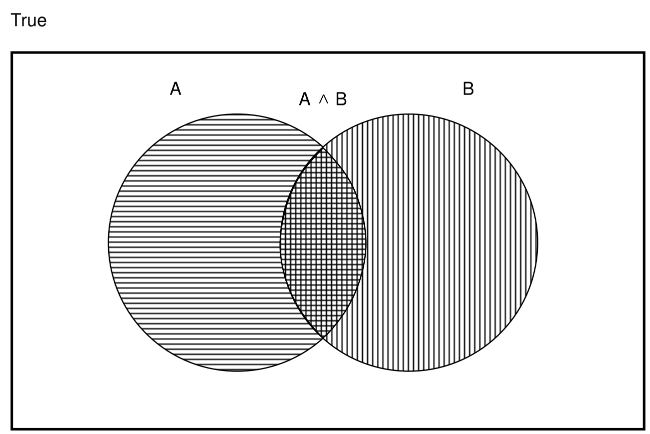
- Necessarily false propositions have probability 0:

$$P(False) = 0$$

- The probability of a disjunction is given by:

$$P(A \vee B) = P(A) + P(B) - P(A \wedge B)$$

Set-theoretic view:



Probability Notations

- Prior Probability (Unconditional probability): P(A)
- Posterior Probability (Conditional probability): P(A | B)

$$P(A|B) \;=\; \frac{P(A \wedge B)}{P(B)} \;\; if P(A) > 0.$$

- Product rule (chain rule):

$$P(A \wedge B) = P(A|B)P(B) = P(B|A)P(A)$$

Thus,

$$\begin{aligned} P(A|B) &= \frac{P(A \wedge B)}{P(B)} \\ P(B|A) &= \frac{P(B \wedge A)}{P(A)} \end{aligned}$$

if $P(A), P(B) > 0.$

- More general form for multivalued random variables:

$$\begin{aligned} P(X = x_i, Y = y_j) &= P(X = x_i|Y = y_j)P(Y = y_j) \\ &= P(Y = y_j|X = x_i)P(X = x_i) \end{aligned}$$

Or simply,

$$P(X, Y) = P(X|Y)P(Y) = P(Y|X)P(X)$$

Bayes' Rule

- Bayes' rule:

$$P(A|E) \;=\; \frac{P(E|A)P(A)}{P(E)}$$

- More general form for multivalued random variables:

$$P(Y = y_j|X = x_i) \;=\; \frac{P(X = x_i|Y = y_j)P(Y = y_j)}{P(X = x_i)}$$

Or simply,

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)}$$

- Conditioning on some background evidence E:

$$P(Y|X, E) = \frac{P(X|Y, E)P(Y|E)}{P(X|E)}$$

Medical Diagnosis Example

- Given:

$$P(Cough|LungCancer) = 0.8$$
$$P(LungCancer) = 0.005$$
$$P(Cough) = 0.05$$

  Find:

$$P(LungCancer|Cough)$$

- Applying Bayes' rule:

$$\begin{aligned} & P(LungCancer|Cough) \\ = \ & \frac{P(Cough|LungCancer)P(LungCancer)}{P(Cough)} \\ = \ & \frac{0.8 \times 0.005}{0.05} = 0.08. \end{aligned}$$

- $P(LungCancer|Cough)$, as diagnostic knowledge, is usually not directly available in domain knowledge. Instead, $P(Cough|LungCancer)$, as causal knowledge, is the more commonly available form.

Prior vs. posterior probabilities

- $P(LungCancer)$ — probability before absorbing evidence. *Prior* probability.
- $P(LungCancer|Cough)$ — probability after absorbing evidence. *Posterior* probability.