Max Flows: Feasible Schedule

Assume *n* roommates $r_1,\ldots r_n$.

For fairness, every day $d_1,\ldots d_n$ a different roommate is supposed to cook dinner.

However, due to other obligations, some roommates are unable to cook on certain days.

Let $C_{i,j}$=true, if r_i can cook on day d_j.

Describe an algorithm to determine if is possible to have a feasible schedule such that each roommate cooks exactly once during the *n* days.

Max Flows: Feasible Schedule

$C_{i,j}$=true, if r_i can cook on day d_j.

Describe an algorithm to determine if is possible to have a feasible schedule such that each roommate cooks exactly once during the n days.

Solution: This is a matching problem.

Create **bipartite graph** in which each roommate $r_1,\ldots r_n$ and each day $d_1,\ldots d_n$ are nodes.

Create an edge (r_i, d_j) iff $C_{i,j}$=true.

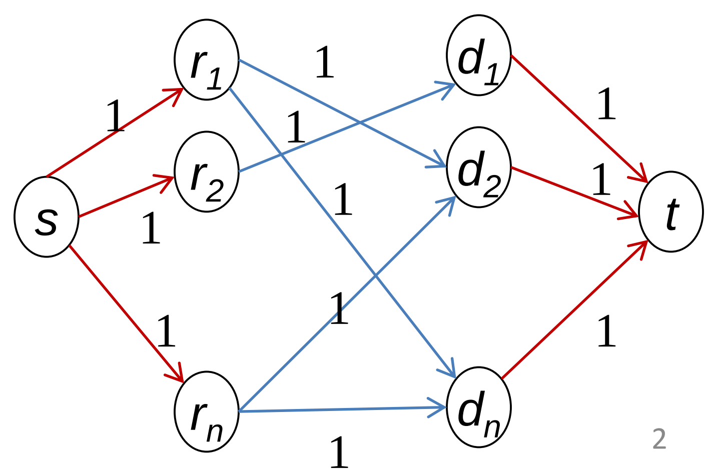
Add source node s with outgoing edges to roommates, and sink t with incoming edges from days. All edge capacities equal 1.

A feasible schedule exists if and only if

The bipartite graph has a perfect matching, i.e.,

A matching touching every vertex.

This happens iff the max s-t flow has value n.



Max Flows: Balanced Assignment

Your company wishes to assign n customers $c_1,\ldots c_n$ to k facilities $f_1,\ldots f_k$.

Each customer can only be served by some facility in his vicinity:
$C_{i,j}$=true means that customer c_i can be served by facility f_j.

An **assignment** of customers to facilities is balanced,
if each facility serves the same number n/k of customers
(assume that n/k is integer).

Given the constraints $C_{i,j}$, describe an algorithm to determine if is possible to construct a balanced assignment

Max Flows: Balanced Assignment

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Given constraints $C_{i,j}$, describe an algorithm to determine if is possible to construct a balanced assignment

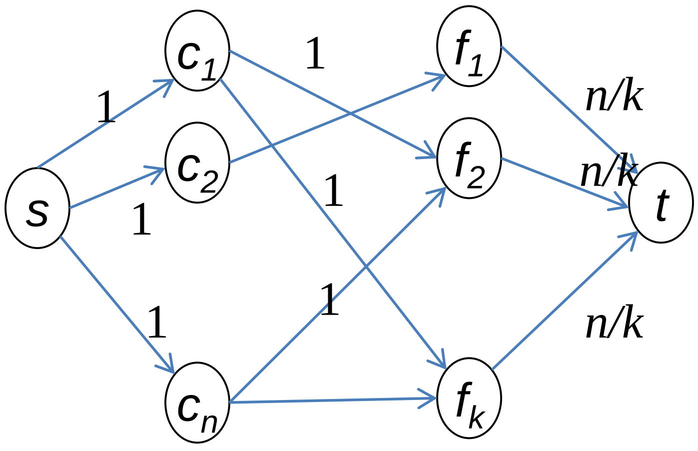
Solution: Create a bipartite graph.

Each customer $c_1,\ldots c_n$ and each facility $f_1,\ldots f_k$ are nodes.

Edge (c_i, f_j) exists iff $C_{i,j}$=true.

Add source s connected to all customers, and sink t with incoming edges from allfacilities. All edge capacities = 1, except for those connecting facilities to t, whose capacity is n/k.

A balanced assignment exists if and only if there is maximum flow of value n.



Max Flows: Constrained Assignment

Your company now wishes to assign n customers $c_1,\ldots c_n$ to k facilities $f_1,\ldots f_k$.

Each customer can only be served by some facility in his vicinity:

$C_{i,j}$=true means that customer c_i can be served by facility f_j

An **assignment** of customers to facilities is constrained, so that facility f_i can serve serves n_i customers where $\sum_{i=1}^{k} n_i = \mathrm{n}$

Describe an algorithm to determine, given the constraints $C_{i,j}$ and the $n_{i,}$ if is possible to construct a constrained assignment that serves all of the customers and, if such an assignment exists, to construct it.

Max Flows: Constrained Assignment

$C_{i,j}$=true means that customer c_i can be served by facility f_j.
Facility f_i serves at most n_i customers where $\sum_{i=1}^{k} n_i = \mathrm{n}$

Describe an algorithm to determine if is possible to construct a constrained assignment given the constraints $C_{i,j}$ and values n_i

Solution: Create a bipartite graph in which each customer $c_1,\ldots c_n$ and each facility $f_1,\ldots f_k$ are nodes.

Edge (c_i, f_j) exists iff $C_{i,j}$=true.

Add source s with outgoing edges to customers, and sink t with incoming edges from all facilities. All edge capacities equal 1, except for those connecting facilities f_i to t, whose capacity is n_i

A constrained assignment exists iff there is maximum flow of value n.