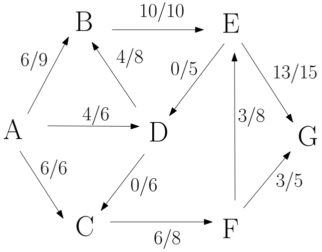
COMP3711H: Design and Analysis of Algorithms

Tutorial 10
Revision of November 21, 2016

HKUST

Question 1

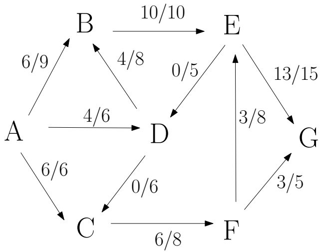
Consider the given graph with flow values f and capacities c (f/c) as shown. $s = A$ and $t = G$.

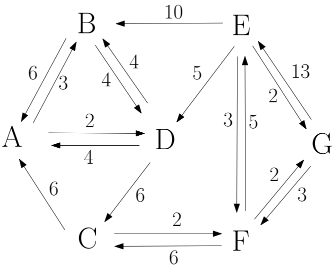


Draw, the residual graph.
Find an augmenting path.
Show the new flow created by adding the augmenting path flow. Is your new flow optimal?
Prove or disprove.

Solution 1

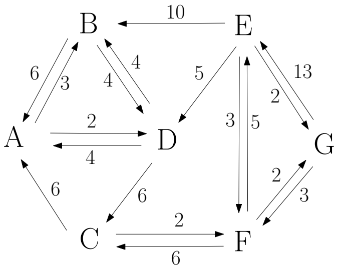
Draw, the residual graph.

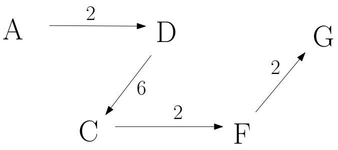




Solution 1

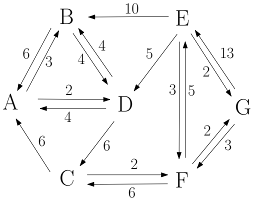
Find an augmenting path.

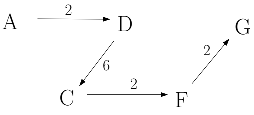


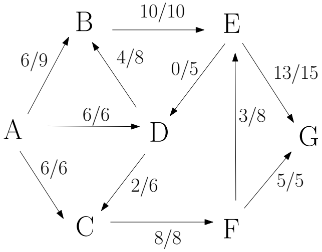


Solution 1

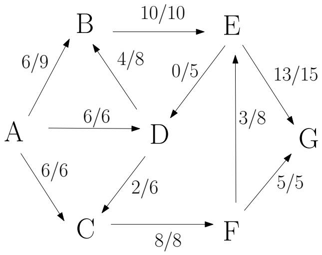
Show the new flow created by adding the augmenting path flow.

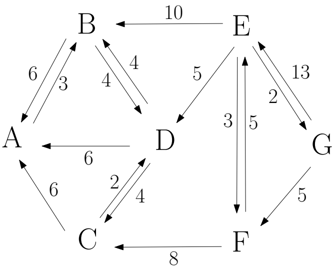






Is your new flow optimal? Prove or disprove.



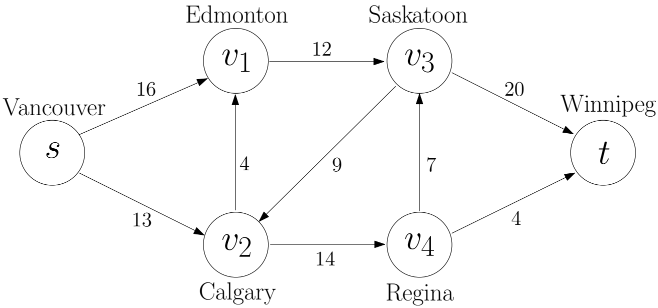


Yes it is. Consider the S, T cut defined by $S = \{A, B, C, D\}$ which is the set of nodes that can be reached from A in the residual graph of the new flow. Then $T = \{E, F, G\}$. Note that the capacity of the cut is $C(S, T) = 10 + 8 = 18$ which is exactly the value of the flow leaving A.

$\Rightarrow$ from the max-flow min-cut theorem, this is a maximal flow.

Question 2

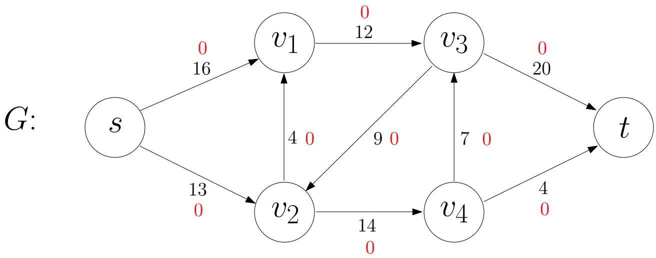
Show the execution of the Edmonds-Karp algorithm on the following flow network.

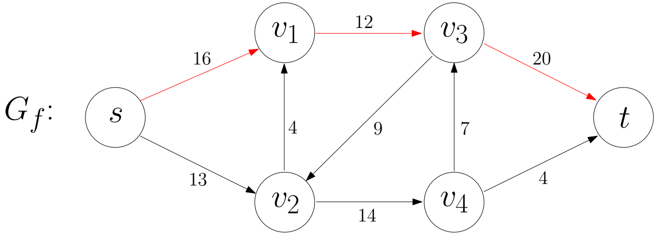


Solution 2

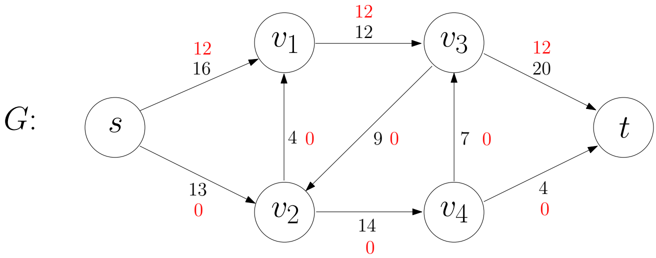
Recall that the *Edmonds-Karp* Algorithm is the version of the Ford-Fulkerson method that always chooses a shortest (by number of edges) Augmenting Path.

Solution 2

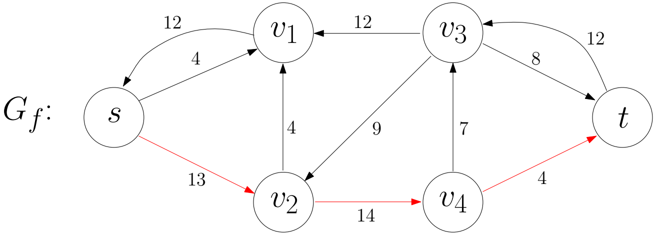

G:
s
v1
v2
v3
v4
t
0
16
13
0
0
12
4 0
9 0
14
0
7 0
0
20
4
0


Gf:
s
v1
v2
v3
v4
t
16
13
12
4
9
14
7
20
4

Solution 2

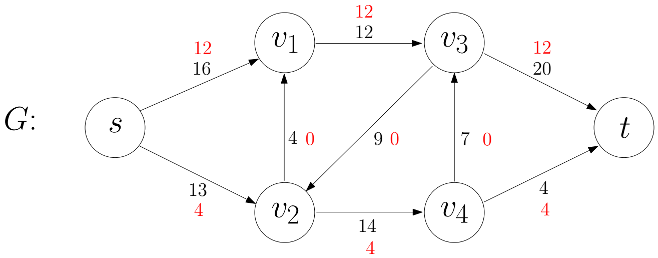


G:
s
v1
v2
v3
v4
t
12
16
13
0
4 0
12
12
9 0
7 0
14
0
12
20
4
0



Gf:
s
v1
v2
v3
v4
t
12
4
13
4
12
9
14
7
8
12
4

Solution 2


G:
s
v1
v2
v3
v4
t
12
16
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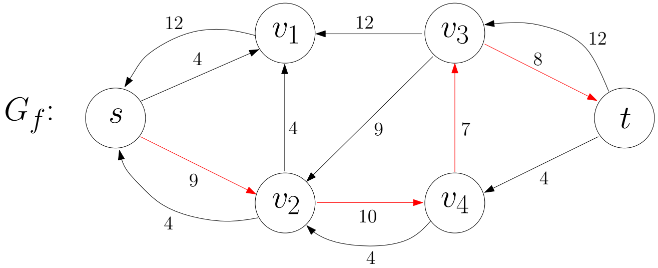
12
20
4 0
9 0
7 0
13
4
14
4
4
4


Gf:
s
v1
v2
v3
v4
t
12
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12
8
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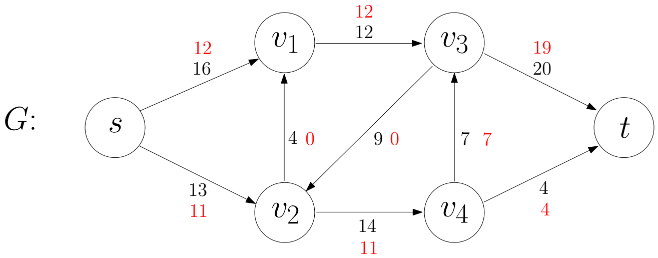
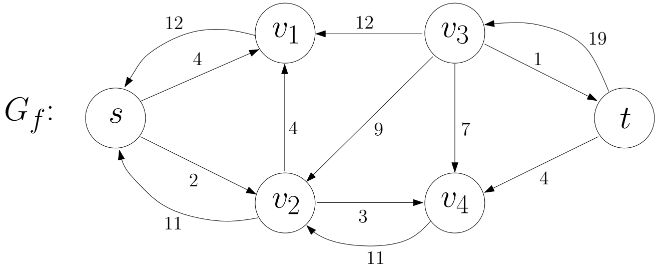
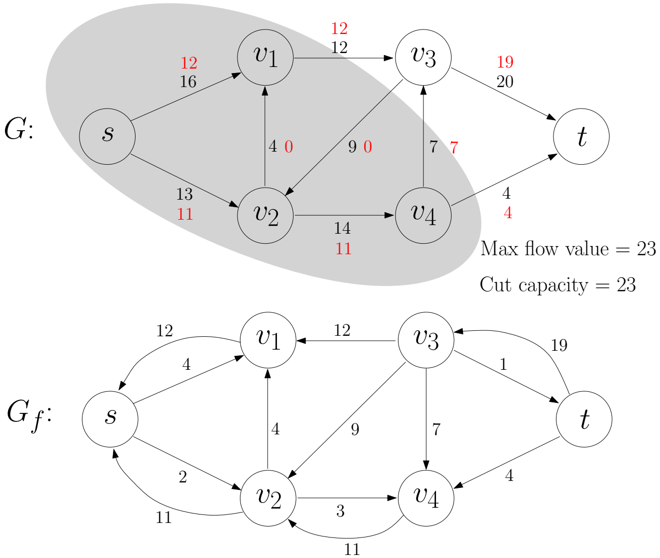
10
4
4

Solution 2


G:
s
v1
v2
v3
v4
t
12
16
13
11
12
12
4 0
9 0
7 7
14
11
19
20
4
4


Gf:
s
v1
v2
v3
v4
t
12
4
2
11
12
4
9
7
3
11
1
19
4

Solution 2


G:
s
v1
v2
v3
v4
t
12
16
13
11
12
12
4 0
9 0
7 7
19
20
14
11
4
4
Max flow value = 23
Cut capacity = 23
Gf:
12
4
2
11
12
4
9
7
1
19
3
11
4