COMP 3711H – Fall 2016
Tutorial 6

1. (CLRS–16.2-4) Professor Midas drives an automobile from Newark to Reno along Interstate 80. His car's gas tank, when full, holds enough gas to travel m miles, and his map gives the distance between gas stations on his route. The professor wishes to make as few gas stops as possible along the way. Give an efficient method by which Professor Midas can determine at which gas stations he should stop and prove that your algorithm yields an optimal solution.

2. Consider the problem of making change for n cents using the fewest number of coins. Assume that each coin's value is an integer.

   (a) Describe a greedy algorithm to make change consisting of quarters (25 cents) , dimes (10) , nickels (5), and pennies (1). Prove that your algorithm yields an optimal solution.

   (b) Suppose that the available coins are in denominations that are powers of c. i.e. the denominations are $c^0, c^1, ..., c^k$ for some integers $c > 1$ and $k \geq 1$. Show that the greedy algorithm always yields an optimal solution.

   (c) Give a set of coin denominations for which the greedy algorithm does not yield an optimal solution. Your set should include a penny so that there is a solution for every value of n.

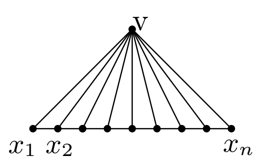
3. Given an undirected graph $G = (V, E)$, its *complement*, $\overline{G}$, is the graph (V, E') such that for all $u \neq v$, $\{u, v\} \in E'$ if and only if $\{u, v\} \notin E$. Prove that either G or $\overline{G}$ is connected.

4. An (undirected) graph $G = (V, E)$ is *bipartite* if there exists some $S \subset V$ such that, for every edge $\{u, v\} \in E$, either (i) $u \in S$, $v \in V - S$ or (ii) $v \in S$, $u \in V - S$.

   Let $G = (V, E)$ be a connected graph. Design an $O(n + e)$ algorithm that checks whether G is bipartite. *Hint: Run BFS.*

   How can you modify your algorithm so that it also works for unconnected graphs?

5.



In the Fan Graph F_n, node v is connected to all the nodes $x_1, \ldots, x_n$, x_1 is also connected to x_2, x_n is also connected to x_{n-1} and every other x_i is connected to x_{i-1} and x_{i+1}. The adjacency lists are given as follows

$$\begin{array}{rrcl} & v & : & x_1, x_2, \ldots, x_n \\ & x_1 & : & v, 2_2 \\ & x_n & : & v, x_{n-1} \\ \forall i \neq 1, n, & x_i & : & v, x_{i-1}, x_{i+1} \end{array}$$

(a) : Describe the tree that is output when BFS is run on F_n starting from initial vertex v? initial vertex x_1? x_n? Other x_i?

(b) : Describe the tree that is output when DFS is run on F_n starting from initial vertex v? initial vertex x_1? x_n? Other x_i?