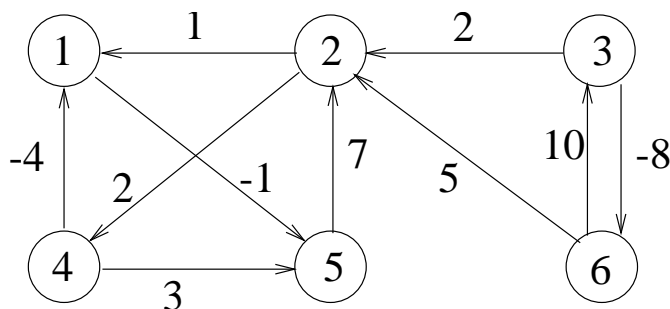


1. Run the Floyd-Warshall algorithm on the weighted, directed graph shown in the figure. Show the matrix  $D^{(k)}$  that results for each iteration of the outer loop.



2. Let  $G = (V, E)$  be Directed Acyclic Graph and  $s$  a vertex from which it's possible to get to all vertices. Show how to build a shortest (i.e., min-cost) path tree rooted at  $s$  in  $O(|V| + |E|)$  time.
3. (CLRS) Give an algorithm that takes as input a directed graph with positive edge weights, and returns the cost of the shortest cycle in the graph (if the graph is acyclic, it should say so). Your algorithm should take time at most  $O(n^3)$ , where  $n$  is the number of vertices in the graph.
4. Assume that all edges have positive weight. Design an algorithm that will, for every pair of vertices, count the *number* of shortest paths between that pair.
5. KFCC is considering opening a series of restaurants along the Highway. The  $n$  available locations are along a straight line; the distances of these locations from the start of the Highway are, in miles and in increasing order:  $m_1, m_2, \dots, m_n$ . The constraints are as follows:
  1. At each location, KFCC may open at most one restaurant.  
 The expected profit from opening a restaurant at location  $i$  is  $p_i$ , where  $p_i > 0$  and  $i = 1, 2, \dots, n$ .
  2. Any two restaurants should be at least  $k$  miles apart,  
 where  $k$  is a given positive integer.

Give a dynamic programming algorithm that determines the locations to open restaurants which maximizes the total expected profit and analyze the running time of your algorithm.