Primal-Dual Weighted Set Cover

v2: Revised 1/11/07

Given universe U
collection of subsets $\mathcal{F}$ of U,
each $F \in \mathcal{F}$ having weight $C(F)$

A collection of subsets *covers* U if their union contains U.
The weight of a cover is sum of the weights of set in cover.

Primal-Dual Weighted Set Cover

v2: Revised 1/11/07

Given universe U
collection of subsets $\mathcal{F}$ of U,
each $F \in \mathcal{F}$ having weight $C(F)$

A collection of subsets *covers* U if their union contains U.
The weight of a cover is sum of the weights of set in cover.

Finding a min-weight cover is NP-Hard. Algorithm below is an f-approximation algorithm, where f is max frequency of element in U (freq(x) is $\#$ of sets to which x belongs.)

Primal-Dual Weighted Set Cover

v2: Revised 1/11/07

Given universe U
collection of subsets $\mathcal{F}$ of U,
each $F \in \mathcal{F}$ having weight $C(F)$

A collection of subsets *covers* U if their union contains U.
The weight of a cover is sum of the weights of set in cover.

Finding a min-weight cover is NP-Hard. Algorithm below is an f-approximation algorithm, where f is max frequency of element in U (freq(x) is $\#$ of sets to which x belongs.)

Primal-Dual Set-Cover

1. Set $\forall F,\ x_F = 0,\ \forall e,\ y_e = 0.$
2. Until all elements are covered do
   Pick an uncovered element e, and increase y_e
   until some set becomes tight.
   Add all newly tight sets to the cover.
   by setting $x_F = 1$ for those sets.
3. Output the cover

Primal-Dual Set-Cover

1. Set $\forall F,\ x_F = 0,\ \forall e,\ y_e = 0.$
2. Until all elements are covered do
   Pick an uncovered element e, and increase y_e
   until some set becomes tight.
   Add all newly tight sets to the cover.
   by setting $x_F = 1$ for those sets.
3. Output the cover

Primal-Dual Set-Cover

1. Set $\forall F,\ x_F = 0,\ \forall e,\ y_e = 0.$
2. Until all elements are covered do
   Pick an uncovered element e, and increase y_e
   until some set becomes tight.
   Add all newly tight sets to the cover.
   by setting $x_F = 1$ for those sets.
3. Output the cover

$$F_1 = \{1, 2, 3, 6\} \qquad C(F_1) = 4$$
$$F_2 = \{2, 4, 5, 8\} \qquad C(F_2) = 6$$
$$F_3 = \{1, 3, 5\} \qquad C(F_3) = 1$$
$$F_4 = \{2, 7, 5\} \qquad C(F_4) = 3$$
$$F_5 = \{1, 7, 8, 6\} \qquad C(F_5) = 9$$
$$F_6 = \{1, 4, 6\} \qquad C(F_6) = 2$$
$$F_7 = \{6, 7, 8\} \qquad C(F_7) = 10$$
$$F_8 = \{2, 4, 6, 7, 8\} \qquad C(F_8) = 7$$

Primal-Dual Set-Cover

1. Set $\forall F,\ x_F = 0,\ \forall e,\ y_e = 0.$
2. Until all elements are covered do
   Pick an uncovered element e, and increase y_e
   until some set becomes tight.
   Add all newly tight sets to the cover.
   by setting $x_F = 1$ for those sets.
3. Output the cover

$$F_1 = \{1, 2, 3, 6\} \qquad C(F_1) = 4$$
$$F_2 = \{2, 4, 5, 8\} \qquad C(F_2) = 6$$
$$F_3 = \{1, 3, 5\} \qquad C(F_3) = 1$$
$$F_4 = \{2, 7, 5\} \qquad C(F_4) = 3$$
$$F_5 = \{1, 7, 8, 6\} \qquad C(F_5) = 9$$
$$F_6 = \{1, 4, 6\} \qquad C(F_6) = 2$$
$$F_7 = \{6, 7, 8\} \qquad C(F_7) = 10$$
$$F_8 = \{2, 4, 6, 7, 8\} \qquad C(F_8) = 7$$

Primal-Dual Set-Cover

1. Set $\forall F,\ x_F = 0,\ \forall e,\ y_e = 0.$
2. Until all elements are covered do
   Pick an uncovered element e, and increase y_e
   until some set becomes tight.
   Add all newly tight sets to the cover.
   by setting $x_F = 1$ for those sets.
3. Output the cover

$F_1 = \{1, 2, 3, 6\}$ $C(F_1) = 4$

$F_2 = \{2, 4, 5, 8\}$ $C(F_2) = 6$

$F_3 = \{1, 3, 5\}$ $C(F_3) = 1$

$F_4 = \{2, 7, 5\}$ $C(F_4) = 3$

$F_5 = \{1, 7, 8, 6\}$ $C(F_5) = 9$

$F_6 = \{1, 4, 6\}$ $C(F_6) = 2$

$F_7 = \{6, 7, 8\}$ $C(F_7) = 10$

$F_8 = \{2, 4, 6, 7, 8\}$ $C(F_8) = 7$

e	y_e	Tight Set(s)	Covered

Primal-Dual Set-Cover

1. Set $\forall F,\ x_F = 0,\ \forall e,\ y_e = 0.$
2. Until all elements are covered do
   Pick an uncovered element e, and increase y_e
   until some set becomes tight.
   Add all newly tight sets to the cover.
   by setting $x_F = 1$ for those sets.
3. Output the cover

$$F_1 = \{1, 2, 3, 6\} \qquad C(F_1) = 4$$
$$F_2 = \{2, 4, 5, 8\} \qquad C(F_2) = 6$$
$$F_3 = \{1, 3, 5\} \qquad C(F_3) = 1$$
$$F_4 = \{2, 7, 5\} \qquad C(F_4) = 3$$
$$F_5 = \{1, 7, 8, 6\} \qquad C(F_5) = 9$$
$$F_6 = \{1, 4, 6\} \qquad C(F_6) = 2$$
$$F_7 = \{6, 7, 8\} \qquad C(F_7) = 10$$
$$F_8 = \{2, 4, 6, 7, 8\} \qquad C(F_8) = 7$$

e	y_e	Tight Set(s)	Covered
1	1	F_3	$1, 3, 5$

Primal-Dual Set-Cover

1. Set $\forall F,\ x_F = 0,\ \forall e,\ y_e = 0.$
2. Until all elements are covered do
   Pick an uncovered element e, and increase y_e
   until some set becomes tight.
   Add all newly tight sets to the cover.
   by setting $x_F = 1$ for those sets.
3. Output the cover

$$F_1 = \{1, 2, 3, 6\} \quad C(F_1) = 4$$
$$F_2 = \{2, 4, 5, 8\} \quad C(F_2) = 6$$
$$F_3 = \{1, 3, 5\} \quad C(F_3) = 1$$
$$F_4 = \{2, 7, 5\} \quad C(F_4) = 3$$
$$F_5 = \{1, 7, 8, 6\} \quad C(F_5) = 9$$
$$F_6 = \{1, 4, 6\} \quad C(F_6) = 2$$
$$F_7 = \{6, 7, 8\} \quad C(F_7) = 10$$
$$F_8 = \{2, 4, 6, 7, 8\} \quad C(F_8) = 7$$

e	y_e	Tight Set(s)	Covered
1	1	F_3	$1, 3, 5$
2	3	F_1, F_4	$2, 6, 7$

Primal-Dual Set-Cover

1. Set $\forall F,\ x_F = 0,\ \forall e,\ y_e = 0.$
2. Until all elements are covered do
   Pick an uncovered element e, and increase y_e
   until some set becomes tight.
   Add all newly tight sets to the cover.
   by setting $x_F = 1$ for those sets.
3. Output the cover

$$F_1 = \{1, 2, 3, 6\} \quad C(F_1) = 4$$
$$F_2 = \{2, 4, 5, 8\} \quad C(F_2) = 6$$
$$F_3 = \{1, 3, 5\} \quad C(F_3) = 1$$
$$F_4 = \{2, 7, 5\} \quad C(F_4) = 3$$
$$F_5 = \{1, 7, 8, 6\} \quad C(F_5) = 9$$
$$F_6 = \{1, 4, 6\} \quad C(F_6) = 2$$
$$F_7 = \{6, 7, 8\} \quad C(F_7) = 10$$
$$F_8 = \{2, 4, 6, 7, 8\} \quad C(F_8) = 7$$

e	y_e	Tight Set(s)	Covered
1	1	F_3	$1, 3, 5$
2	3	F_1, F_4	$2, 6, 7$
4	1	F_6	4

Primal-Dual Set-Cover

1. Set $\forall F,\ x_F = 0,\ \forall e,\ y_e = 0.$
2. Until all elements are covered do
   Pick an uncovered element e, and increase y_e
   until some set becomes tight.
   Add all newly tight sets to the cover.
   by setting $x_F = 1$ for those sets.
3. Output the cover

$$F_1 = \{1, 2, 3, 6\} \quad C(F_1) = 4$$
$$F_2 = \{2, 4, 5, 8\} \quad C(F_2) = 6$$
$$F_3 = \{1, 3, 5\} \quad C(F_3) = 1$$
$$F_4 = \{2, 7, 5\} \quad C(F_4) = 3$$
$$F_5 = \{1, 7, 8, 6\} \quad C(F_5) = 9$$
$$F_6 = \{1, 4, 6\} \quad C(F_6) = 2$$
$$F_7 = \{6, 7, 8\} \quad C(F_7) = 10$$
$$F_8 = \{2, 4, 6, 7, 8\} \quad C(F_8) = 7$$

e	y_e	Tight Set(s)	Covered
1	1	F_3	$1, 3, 5$
2	3	F_1, F_4	$2, 6, 7$
4	1	F_6	4
8	2	F_2	8

Primal-Dual Set-Cover

1. Set $\forall F,\ x_F = 0,\ \forall e,\ y_e = 0.$
2. Until all elements are covered do
   Pick an uncovered element e, and increase y_e
   until some set becomes tight.
   Add all newly tight sets to the cover.
   by setting $x_F = 1$ for those sets.
3. Output the cover

$F_1 = \{1, 2, 3, 6\}$ $C(F_1) = 4$

$F_2 = \{2, 4, 5, 8\}$ $C(F_2) = 6$

$F_3 = \{1, 3, 5\}$ $C(F_3) = 1$

$F_4 = \{2, 7, 5\}$ $C(F_4) = 3$

$F_5 = \{1, 7, 8, 6\}$ $C(F_5) = 9$

$F_6 = \{1, 4, 6\}$ $C(F_6) = 2$

$F_7 = \{6, 7, 8\}$ $C(F_7) = 10$

$F_8 = \{2, 4, 6, 7, 8\}$ $C(F_8) = 7$

e	y_e	Tight Set(s)	Covered
1	1	F_3	$1, 3, 5$
2	3	F_1, F_4	$2, 6, 7$
4	1	F_6	4
8	2	F_2	8

Cover found is

F_1, F_2, F_3, F_4, F_6