Abstract

The max-min fair allocation problem, also known as the Santa Claus problem, is a fundamental problem in combinatorial optimization. Given a set of players P, a set of indivisible resources R, and a set of non-negative values $\{v_{pr}\}_{p\in P,r\in R}$, an allocation is a partition of R into disjoint subsets $\{C_p\}_{p\in P}$ so that each player p is assigned the resources in C_p. We hope to find a fair allocation of resources to players so that the welfare of the least lucky player is maximized. More precisely, we want to maximize $\min_p \sum_{r\in C_p} v_{pr}$. In the restricted case of this problem, we have $v_{pr} \in \{v_r, 0\}$. That is, each resource r has an intrinsic value v_r, and it is worth value v_r to those players who desire it and value 0 to those who do not.

This thesis investigates the restricted case of the problem, namely, the restricted max-min fair allocation. We improve the upper bound for the integrality gap of a well-known configuration LP which finds application in estimating the optimal value of the problem. We also propose two polynomial-time approximation algorithms that improve upon the state-of-the-art approximation ratio. With regard to this problem, the configuration LP is the only LP relaxation known to have a constant integrality gap, and plays an important role in estimating and approximating the problem. Our first result is an improvement of the upper bound for the integrality gap from 4 to $3\frac{21}{26}$. It is obtained by giving a tighter analysis of a local search algorithm in the literature. Hence, by solving the configuration LP, we can estimate the optimal value of the problem within a factor of $3\frac{21}{26}$. However, as the local search is not known to run in polynomial time, our first result does not lead to an efficient approximation algorithm that returns some allocation. An open question is whether one can find an efficient approximation algorithm whose approximation ratio matches the bound for the integrality gap. Prior to our work, the best known approximation ratio is $6 + 2\sqrt{3} + \delta$ where δ is an arbitrarily small constant. Our first algorithm achieves an approximation ratio of $6 + \delta$. The algorithm and its analysis are purely combinatorial. Our second algorithm can be regarded as a generalization of the original local search used in the integrality gap result. It achieves a trade-off between the running time and the quality of solutions. It turns out that by relaxing the ratio slightly to $4 + \delta$, we can achieve a polynomial running time.