ABSTRACT

Planar point location is a classical problem in computational geometry. Several point location algorithms are known that achieve optimal worst-case query time. However, there are still a lot of research on this topic. In many applications, certain regions in a planar subdivision are more frequently queried. This raises the question of whether a better query time can be obtained by exploiting the query distribution. In this thesis, we study the point location algorithms that exploit the query distribution to acheive a better query time. We study both the static and dynamic cases. In the static case, most of the existing solutions assume that the query distribution is fixed and can be accessed quickly. In the scenario that the query distribution is not known in advance, we propose a point location algorithm for convex and connected subdivisions with total running times $O(\mathrm{OPT}+n)$ and $O(\mathrm{OPT} + n + |\sigma| \log(\log^* n))$, respectively, to process any online query sequence σ. The quantity OPT is the minimum time to answer queries in σ by any algorithm that can be modeled as a linear decision tree. In the dynamic case, nothing is known about how to exploit the query distribution in answering point location queries. Suppose that there is a fixed query distribution in $\mathbb{R}^2$, and we are given an oracle that can return in $O(1)$ time the probability of a query point falling into a polygonal region of constant complexity. We can maintain a convex/connected subdivision $\mathcal{S}$ with n vertices such that a query is answered in $O(\mathrm{opt})$ expected time for a convex subdivision and $O(\mathrm{opt}+\log(\log^* n))$ expected time for a connected subdivision. The quantity opt is the minimum expected time of the best linear decision tree for point location in $\mathcal{S}$. The space and construction time are $O(n \log^2 n)$. An update of $\mathcal{S}$ as a mixed sequence of k edge insertions and deletions takes $O(k \log^5 n)$ amortized time for a convex subdivision and $O(k \log^8 n + \log^{31} n)$ for a connected subdivision. As a corollary, the randomized incremental construction of the Voronoi diagram of n sites can be performed in $O(n \log^5 n)$ expected time so that, during the incremental construction, a nearest neighbor query can be answered optimally with respect to the intermediate Voronoi diagram at that time.