HKUST

Sample Questions of Mathematics Placement Test

(Multiple Choice Questions Only) Name:_____

Student ID:_____

Duration: 120 minutes Department:_____

Directions:

- All mobile phones and pagers should be switched off during the examination.
- Please write your Name, ID number, and Department in the space provided above.
- No graphical calculators are allowed.
- Once you are allowed to start the test, please check that you have 11 pages including this page.
- This test will be divided into two parts: Part A and Part B according to the difficulty of questions. Please write all your answers on the given **Bubble Answer Sheet**.

Please answer ALL of the following multiple choice questions.

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1. What mathematics background do you have ?

   (a) HKAL Pure Math (b) HKAS Math & Statistic (c) GCE A-level

   (d) IB Math Methods SL or above (e) Others

2. Which of the following departments/programs you belong to ?

   (a) BICH / BIOL (b) CHEM (c) PHYS (d) MATH (e) None of the previous

3. Which of the following departments/programs you belong to ?

   (a) CENG (b) CIVL (c) COMP (d) CPEG (e) None of the previous

4. Which of the following departments/programs you belong to ?

   (a) ELEC (b) MECH (c) IELM (d) BBA (e) None of the previous

5. Which of the following departments/programs you belong to ?

   (a) ACCT (b) ECON (c) QFIN (d) GBUS (e) None of the previous

Part A: This part will include simple problems in elementary mathematics.

6. Four iced lemon tea cost \$10. How much does it cost for seven iced lattes?

   (a) 16.5 (b) 17 (c) 17.5 (d) 18 (e) 19.5

7. If $9y = 3^{2+x}$, then $y =$

   (a) 3^x (b) $3x$ (c) 3 (d) 9 (e) 27

8. Given that $5a^2 - 4a - 13 - 3(a^2 + a + 2) = 0$. What is the sum of the possible values of a?

   (a) 3 (b) 3.5 (c) 4 (d) 4.5 (e) 5

9. If $4x^2 = 25(y^2 + 2)$, then $\dfrac{50}{2x - 5y} =$

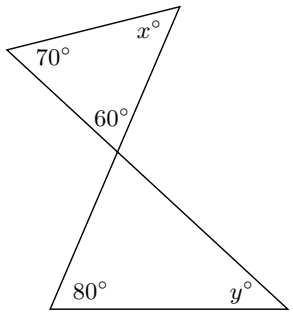
   (a) $x + y$ (b) $2x - 5y$ (c) $2x + 5y$ (d) $x - y$ (e) x

10. If $2x^2 - 30 > 2$, what are the possible values of x?

    (a) $x > 4$ (b) $x < 4$ (c) $x > -4$

    (d) $-4 < x < 4$ (e) $x < -4$ or $x > 4$

11. From the figure below, $x + y =$



    (a) 40 (b) 60 (c) 90 (d) 120 (e) 150

12. A threater charges \$10 for children and \$14 for adults to attend a drama. A total of 1,600 tickets were sold for a total of \$18,400. How many children's ticket were sold?

(a) 600 (b) 700 (c) 800 (d) 900 (e) 1,000

13. Three coins are tossed at the same time. What is the probability that exactly two heads are face down?

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{3}{8}$ (d) $\frac{1}{2}$ (e) $\frac{5}{8}$

14. John worked 4 less than twice as many hours as Betty. Which of the following choices represents the number of hours that John, J, worked based on the number of hours that Betty, B, worked?

(a) $J = 2B - 4$ (b) $J = 2 - 4B$ (c) $J = 2B + 4$
(d) $J = 4 - 2B$ (e) $J = -2B - 4$

15. The domain of $f(x) = \dfrac{x^3 + x - 1}{x^2 + 4}$ is

(a) all $x \neq 2$ (b) all $x \neq -2$ (c) all $x \neq 2, -2$
(d) $x \geqslant 2$ (e) all reals

16. The roots of the equation $f(x) = 0$ are 1 and -2. The roots of $f(-3x) = 0$ are

(a) 1 and -2 (b) $-\frac{1}{3}$ and $\frac{2}{3}$ (c) $-\frac{1}{3}$ and -1
(d) -3 and 6 (e) 3 and -6

17. The set of x-intercepts of the graph of $f(x) = 2x^3 - 4x^2 - 2x + 4$ is

(a) $\{1\}$ (b) $\{-1, 1\}$ (c) $\{1, 2\}$
(d) $\{-1, 1, 2\}$ (e) $\{-1, -2, 2\}$

18. In the xy-coordinate plane, line k passes through the point $(4, 1)$ and is perpendicular to the graph of the line whose equation is $3y = 2x - 18$. Which of the following is the equation for line k?

(a) $2y = 3x - 12$ (b) $2y = -3x + 14$ (c) $3y = 2x + 21$

(d) $3y = -2x + 21$ (e) $2y = -3x - 12$

19. $f(x) = x^2 + \frac{1}{2}x + k$ is the equation of a quadratic function with a graph that passes through the x-axis when $x = 3$. What is the value of k?

(a) $\frac{21}{2}$ (b) 9 (c) $-\frac{21}{2}$ (d) -9 (e) -21

20. The function $f(x) = x^3 + \frac{1}{2}x - \frac{5}{2}$ has exactly one real zero. It is between

(a) -2 and -1 (b) -1 and 0 (c) 0 and 1 (d) 1 and 2 (e) 2 and 3

21. The period of $f(x) = \cos\frac{2\pi}{3}x$ is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) 3 (e) 6

22. If $f^{-1}(x)$ is the inverse of $f(x) = 3e^{-x}$, then $f^{-1}(x) =$

(a) $\ln\left(\frac{3}{x}\right)$ (b) $\ln\left(\frac{x}{3}\right)$ (c) $\left(\frac{1}{3}\right)\ln x$ (d) $\sqrt{\ln x}$ (e) $\ln(3 - x)$

23. Solve $\log_x 2 + \log_x 3 = \frac{1}{2}$

(a) $x = 36$ (b) $x = 25$ (c) $x = 6$ (d) $x = \sqrt{6}$ (e) $x = 5$

Part B: The following questions are more advanced Calculus problems.

24. $\lim_{x\to 0} \frac{\tan 4x}{5x} =$

(a) $\frac{1}{5}$ (b) $\frac{4}{5}$ (c) 0 (d) $-\frac{2}{5}$ (e) does not exist

25. Let f be the function defined below:

$$f(x) = \begin{cases} 1-x, & \text{if } -1 \leqslant x < 0, \\ 2x^2 - 3, & \text{if } 0 \leqslant x < 1, \\ -x + 3, & \text{if } 1 < x < 3, \\ 1, & \text{if } x = 3, \\ 2x - 6, & \text{if } 3 < x \leqslant 4. \end{cases}$$

Then $\lim_{x \to 3} f(x)$

(a) equals 0 (b) equals 1 (c) equals 2 (d) does not exist (e) none of these

26. For which values of k is the following function continuous at $x = 4$?

$$f(x) = \begin{cases} \cos \dfrac{\pi}{x}, & \text{if } x \leqslant 4, \\ k\sqrt{x}, & \text{if } x > 4. \end{cases}$$

(a) $k = \dfrac{1}{\sqrt{2}}$ (b) $k = \dfrac{1}{2\sqrt{2}}$ (c) $k = -\dfrac{1}{\sqrt{2}}$ (d) $k = \dfrac{1}{4}$ (e) $k = -\dfrac{1}{4}$

27. Suppose $f(1) = \dfrac{2}{3}$ and $f(2) = -\dfrac{1}{3}$. Which of the following statements is always true?

(a) If f is continuous on $[1, 2]$, then the maximum value of f on $[1, 2]$ is less than $\dfrac{2}{3}$.

(b) f takes on maximum value on $[1, 2]$.

(c) If f is continuous, then f has a zero on $[1, 2]$.

(d) If f is continuous on $[1, 2]$, then f takes on a mimimum value value on $(1, 2)$.

(e) If f is not continuous on $[1, 2]$, then f does not take on a mimimum value on $[1, 2]$.

28. Solve $\dfrac{x^2 - x - 6}{x^2 + 4x - 5} \geq 0$.

(a) $-\infty < x < -5$ or $-2 \leq x < 1$ or $3 \leq x < \infty$

(b) $-5 < x < -2$ or $1 < x < 3$

(c) $-\infty < x < -2$ or $1 < x < 3$ or $3 < x < \infty$

(d) $-2 \leq x < 3$

(e) $-5 < x < -2$ or $3 \leq x < \infty$

29. Which if the following expressions gives the derivative of $f(x) = \sin 2x$?

(a) $f'(x) = \lim_{x\to 0} \dfrac{\sin 2x}{2x}$

(b) $f'(x) = \lim_{x\to 0} \dfrac{\sin 2(x+h) - \sin 2x}{2x}$

(c) $f'(x) = \lim_{h\to 0} \dfrac{\sin 2(x+h) - \sin 2x}{h}$

(d) $f'(x) = \lim_{h\to 0} \dfrac{\sin(2x+h) - \sin 2x}{h}$

(e) $f'(x) = \lim_{h\to x} \dfrac{\sin 2(x+h) - \sin 2x}{h}$

30. Suppose $f(x)$ is a differentiable function with $f(1) = 2$, $f(2) = -2$, $f'(2) = 5$, $f'(1) = 3$ and $f(5) = 1$. An equation of a line is a tangent to the graph of f is

(a) $y - 3 = 2(x-1)$ (b) $y - 2 = x - 1$ (c) $y - 2 = 5(x-1)$

(d) $y + 4 = 5(x-2)$ (e) $y - 4 = 5(x-2)$

31. The derivative of $y = \dfrac{2-x}{6x+2}$ is

(a) $-\dfrac{7}{2(3x+1)^2}$ (b) $\dfrac{6x-5}{2(3x+1)^2}$ (c) $-\dfrac{9}{2(3x+1)^2}$

(d) $\dfrac{7}{2(3x+1)^2}$ (e) $\dfrac{7-6x}{2(3x+1)^2}$

32. The derivative of $y = \dfrac{2(e^{-x} - e^x)}{e^x + e^{-x}}$ is

(a) 0 (b) 1 (c) $\dfrac{-4}{(e^x + e^{-x})^2}$ (d) $\dfrac{-8}{(e^x + e^{-x})^2}$ (e) $\dfrac{-2}{e^{2x} + e^{-2x}}$

33. The derivative of $y = \dfrac{-x}{\sqrt{1-x^2}}$ is

(a) $\dfrac{2x^2 - 1}{(1-x^2)^{3/2}}$ (b) $\dfrac{1}{x^2 - 1}$ (c) $\dfrac{-1}{\sqrt{1-x^2}}$

(d) $\dfrac{2x^2 - 1}{(1-x^2)^{1/2}}$ (e) none of these

34. The derivative $\dfrac{dy}{dx}$ of $x^3 - 2xy + y^3 = 1$ is

(a) $\dfrac{3x^2}{2x - 3y^2}$ (b) $\dfrac{3x^2 - 2}{2 - 3y^2}$ (c) $\dfrac{2y - 3x^2}{3y^2 - 2x}$

(d) $\dfrac{3x^2 + 3y^2 - 2y}{x}$ (e) $\dfrac{3x^2 + 3y^2}{2x}$

35. If $f(x) = 16x^{1/2}$, then $f''(9)$ is equal to

(a) $-\dfrac{4}{9}$ (b) -16 (c) $-\dfrac{4}{3}$ (d) -3 (e) $-\dfrac{4}{27}$

36. The limit $\lim\limits_{x \to 0} \dfrac{\cos x - 1}{2x}$ is

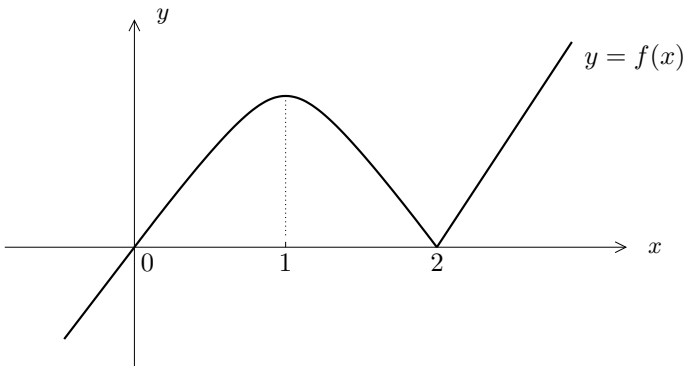
(a) nonexistent (b) $-\dfrac{1}{2}$ (c) -1 (d) ∞ (e) none of these

37. From the values of f shown, estimate $f'(2)$.

x	1.92	1.94	1.96	1.98	2.00
$f(x)$	6.00	5.00	5.40	5.10	5.00

(a) -0.10 (b) -0.20 (c) -5 (d) -10 (e) -25

38. Which of the following is true about the graph of the function $f(x)$?



(a) $f'(0) < 0$, $f'(1) = 0$, and $f'(2) > 0$

(b) $f'(0) > 0$, $f''(1) > 0$, and $f'(2)$ does not exist

(c) $f'(0) > 0$, $f'(1) = 0$, and $f'(2) = 0$

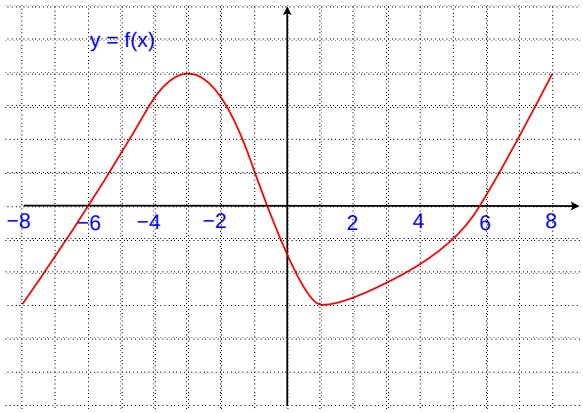
(d) $f'(0) > 0$, $f''(1) < 0$, and $f'(2)$ does not exist

(e) $f'(0) > 0$, $f'(1) = 0$, and $f'(2) = \infty$

39. Find the point of inflection of $g(x) = x^2 - \dfrac{27}{x}$, $x > 0$.

(a) 1 (b) 3 (c) 9 (d) 27 (e) -3

40. The derivative of the following differentiable function is always negative on the following intervals:



(a) $-2 < x < 0$ (b) $-8 < x < -6$ (c) $0 < x < 6$ (d) $2 < x < 8$ (e) $6 < x < 8$

41. The lengths of the sides of a square are increasing at a constant rate of 2 ft/min. In term of the perimeter, P, what is the rate of change of the area of the square in square feet per minute?

(a) P (b) $-P$ (c) $2P$ (d) $4P$ (e) $8P$

42. The indefinite integral $\displaystyle\int \left(x - \frac{1}{3x}\right)^2 dx =$

(a) $\dfrac{1}{3}\left(x - \dfrac{1}{3x}\right)^3 + C$ (b) $x^2 - \dfrac{2}{3} + \dfrac{1}{9x^2} + C$ (c) $\dfrac{x^3}{3} - 2x - \dfrac{1}{9x} + C$

(d) $\dfrac{x^3}{2} - \dfrac{2}{3}x - \dfrac{9}{x} + C$ (e) none of these

43. The definite integral $\displaystyle\int_2^5 \frac{dt}{\sqrt{6-t}} =$

(a) 1 (b) -2 (c) 4 (d) -1 (e) 2

44. For $x > 0$, compute $\displaystyle\frac{d}{dx}\left(\int_x^{x^4} t^{1/2}\,dt\right)$.

(a) $x^2 - \sqrt{x}$ (b) $4x^5 - \sqrt{x}$ (c) $4x^4 - \sqrt{x}$ (d) $\displaystyle\frac{1}{x} - \frac{1}{2\sqrt{x}}$ (e) $\displaystyle-\frac{1}{2x} + \frac{1}{2\sqrt{x}}$

45. If $f(x)$ is continuous on the interval $1 \leqslant x \leqslant 2$ and $1 < c < 2$, then $\displaystyle\int_c^2 f(x)\,dx$ is equal to

(a) $\int_1^c f(x)\,dx + \int_c^2 f(x)\,dx$ (b) $\int_1^c f(x)\,dx - \int_1^2 f(x)\,dx$ (c) $\int_c^1 f(x)\,dx + \int_2^1 f(x)\,dx$

(d) $\int_1^2 f(x)\,dx - \int_1^c f(x)\,dx$ (e) $\int_1^c f(x)\,dx - \int_2^c f(x)\,dx$

46. $\displaystyle\int \sin 2\theta \cos 2\theta\,d\theta =$

(a) $\displaystyle-\frac{\sin^2 2\theta}{4} + C$ (b) $\displaystyle-\frac{1}{8}\cos 4\theta + C$ (c) $\displaystyle\frac{\cos^2 2\theta}{4} + C$

(d) $\displaystyle\frac{1}{4}\sin 4\theta + C$ (e) $\cos 4\theta + C$

47. $\displaystyle\int \frac{\sin x\,dx}{\sqrt{1+\cos x}} =$

(a) $\displaystyle\frac{1}{2}(1+\cos x)^{1/2} + C$ (b) $-\ln\sqrt{1+\cos x} + C$ (c) $-2\sqrt{1+\cos x} + C$

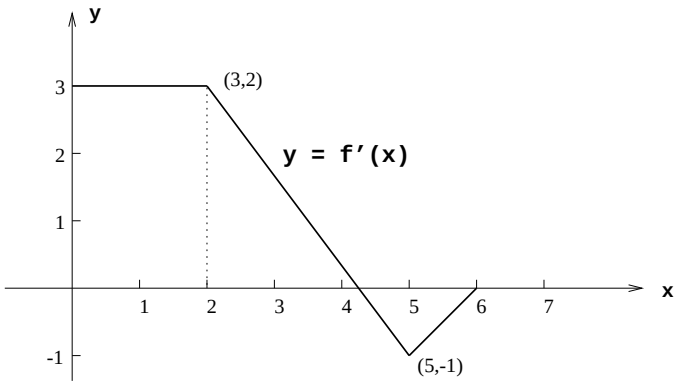
(d) $-\ln|1+\cos x| + C$ (e) $\displaystyle-\frac{2}{3(1+\cos x)^{3/2}} + C$

48. $\displaystyle\int 2x\cos x\,dx =$

(a) $2x\sin x + C$ (b) $2x\sin x + 2\cos x + C$ (c) $2x\sin x + \cos x + C$

(d) $2\cos x - x\sin x + C$ (e) $\displaystyle\frac{x^2}{2}\sin x + C$

49. The graph of the derivative of f, f' is shown below. If $f(0) = 7$, find $f(6)$.



(a) 9 (b) 11 (c) 12 (d) 14 (e) 15

50. $\displaystyle\int_{-\infty}^{0} e^x\,dx =$

(a) 1 (b) $\dfrac{1}{e}$ (c) -1 (d) $-\dfrac{1}{e}$ (e) none of these

51. Let $\vec{a} = 3\vec{i} + 4\vec{j}$ and $|\vec{b}| = 4$. The angle between $\vec{a}$ and $\vec{b}$ is 60°. Then $\vec{a} \cdot \vec{b} =$

(a) $\dfrac{5}{\sqrt{3}}$ (b) 10 (c) 20

(d) -20 (e) -10

52. Which sequence converges?

(a) $a_n = n - \dfrac{4}{n}$ (b) $a_n = -1 + \dfrac{(-1)^{n^2}}{n^2}$ (c) $a_n = \cos\dfrac{n\pi}{2}$

(d) $a_n = \dfrac{n!}{3^n}$ (e) $a_n = \dfrac{n^2}{\ln n}$

53. The n-term of an arithmetic sequence is $4+10n$. Find the sum of the first 100 terms of the sequence.

(a) 1004 (b) 24900 (c) 50400

(d) 50900 (e) 51400

54. Note that $\dfrac{1}{n(n+1)} = \dfrac{1}{n} - \dfrac{1}{n+1}$ $(n \geqslant 1)$. Then $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n(n+1)}$ equals

(a) 0 (b) 2 (c) 3 (d) $\dfrac{3}{2}$ (e) ∞

55. The sum of the geometric series $1 - \dfrac{1}{2} + \dfrac{1}{4} - \dfrac{1}{8} + \dfrac{1}{16} - \cdots$ is

(a) $\dfrac{2}{3}$ (b) $\dfrac{5}{8}$ (c) $\dfrac{1}{2}$ (d) $\dfrac{3}{4}$ (e) $\dfrac{3}{8}$