Open List Proportional Representation

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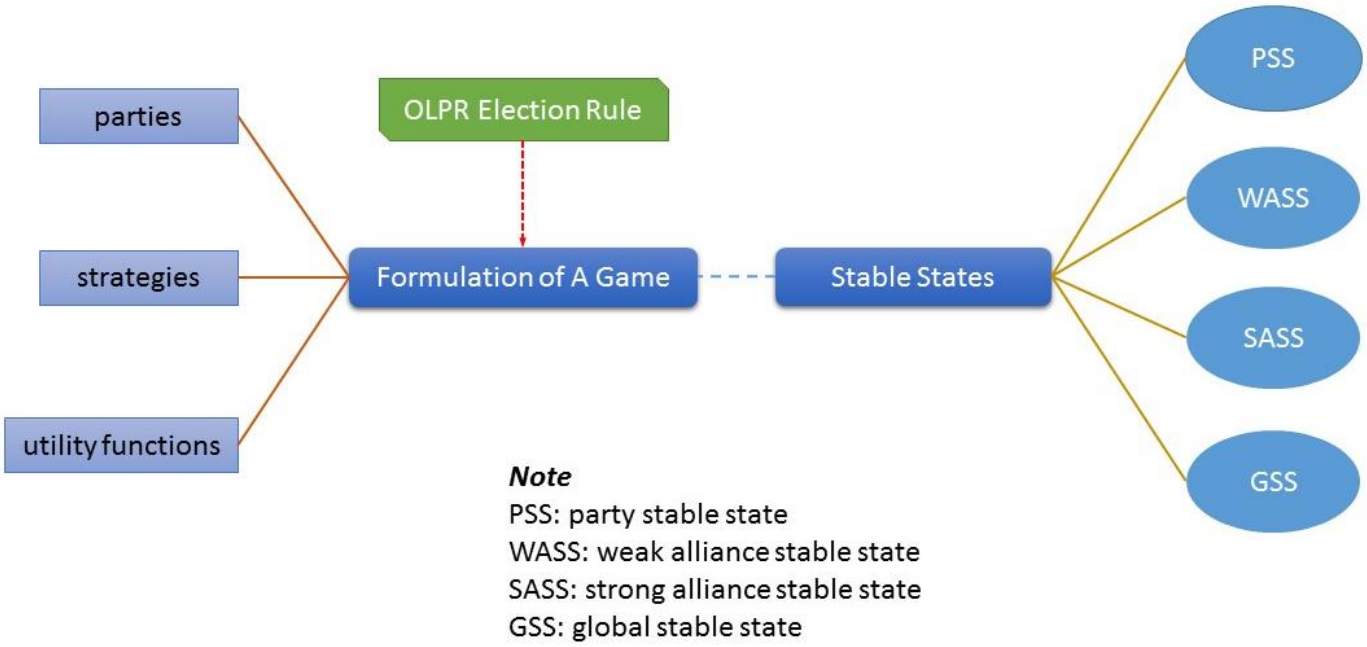
Introduction

Proportional representation (PR) is a principle applied to voting systems such that the number of seats won by a party or a set of candidates is proportional to the number of votes that party or that set of candidates receives. Open list proportional representation (OLPR), a subtype of PR, was used in the 2012 Hong Kong Legislative Council Geographical Constituencies (HKLCGC) election. This project aims to formulate an OLPR election in one constituency into a strategic game and explore possible stable states that parties might prefer to achieve. Existence properties of these stable states are studied.

Overview

The high level structure is given in the following diagram.



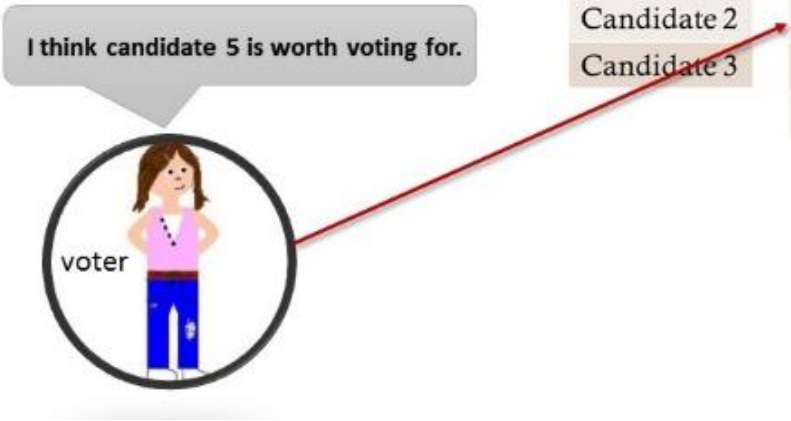
Based on the OLPR election rule, a strategic game is formulated by considering its three essential components:

- Parties
- Strategies
- Utility functions

Four stable states (PSS, WASS, SASS, and GSS) are proposed for the election game. Subsequently, their existence properties are studied.

OLPR Election Rule

In each constituency, voters are presented with a set of candidate lists, any two of which share no common candidate. Each voter is assumed to have one vote and cast her vote to the list containing her desired candidate. At the same time, candidates are assumed to be affiliated with parties. Thus, behind the stage, parties are competing to secure more seats in legislative council. After voting, winners are produced in possibly two rounds. The election rule is illustrated by the following example.



List 1
Candidate 1
Candidate 2
Candidate 3

List 2	
Candidate 4	
Candidate 5	+1
Candidate 6	
Candidate 7	

List 3
Candidate 8
Candidate 9

After voters cast their votes, there is a vote count for each candidate (marked on the right of each candidate). Vote count of each list is the sum of vote counts of candidates in it (marked on the right of list identifier).

List 1	18
Candidate 1	3
Candidate 2	7
Candidate 3	8

List 2	17
Candidate 4	5
Candidate 5	4
Candidate 6	7
Candidate 7	1

List 3	12
Candidate 8	10
Candidate 9	2

In total, there are 47 voters, 9 candidates. Suppose that 3 seats are up for election. Then the quota of the constituency $Q = ceil\left(\frac{47}{3}\right) = 16$.

List 1	18
Candidate 1	
Candidate 2	
Candidate 3	

List 2	17
Candidate 4	
Candidate 5	
Candidate 6	
Candidate 7	

List 3	12
Candidate 8	
Candidate 9	

1st round:

- List 1 produces $\text{floor}(\frac{18}{16}) = 1$ winner: candidate 1.
- List 2 produces $\text{floor}(\frac{17}{16}) = 1$ winner: candidate 4.
- List 3 produces $\text{floor}(\frac{12}{16}) = 0$ winner.

After 1st round, there is 1 seat left unallocated and the lists become

List 1	2
Candidate 2	
Candidate 3	

List 2	1
Candidate 5	
Candidate 6	
Candidate 7	

List 3	12
Candidate 8	
Candidate 9	

2nd round: the one unallocated seat is given to a list with highest votes left from 1st round

- List 3 produces a winner: candidate 8.

Formulation of Games

Parties	They can be viewed as numbers from 1 to n.
Strategies	For a party, its strategy consists of two parts: ▪ with whom to form an alliance or act independently, ▪ how to arrange candidate lists with allies or on its own. The set of strategies for party i is $\boldsymbol{\mathcal{X}}_i$.
Utility functions	Utility functions are constructed with the help of a vote count function.

An n-party OLPR election game is assembled as a tuple $(\boldsymbol{\mathcal{X}}, \pi, \boldsymbol{u})$, where

- $\boldsymbol{\mathcal{X}} = \boldsymbol{\mathcal{X}}_1 \times \ldots \times \boldsymbol{\mathcal{X}}_n$,
- π is a vote count function that gives each candidate a number of votes,
- $\boldsymbol{u} = (u_1, \ldots, u_n)$, with each u_i the utility function for party i.

Stable States

Name	Description	Existence in a game	Existence given an alliance structure
Party stable state (PSS)	Given others' strategies fixed, any party cannot gain more seats by changing its own strategy	Yes	Hypothesis 1
Weak alliance stable state (WASS)	Given others' strategies fixed, any party as well as its allies cannot gain more seats by changing its own strategy	Yes	Hypothesis 2
Strong alliance stable state (SASS)	Given strategies of parties in other alliances fixed, any party cannot gain more seats by changing strategies of parties in its own alliance	Yes	Not necessarily
Global stable state (GSS)	Any party cannot gain more seats by changing strategies of parties in the game (possibly changing alliances)	Hardly, only in special cases	Yes if there is one

Hypothesis 1: Given an alliance structure in a game, there is always a party stable state with respect to it.

Hypothesis 2: Given an alliance structure in a game, there is always a weak alliance stable state with respect to it.

These two hypotheses are hard to prove and only verified in a subspace of the entire problem space by a testing program.

Conclusion

This project has formulated an n-party OLPR election game in strategic form, for which a range of stable states have been defined. Among these stable states are party stable state, weak alliance stable state, strong alliance stable state, and global stable state. In an n-party OLPR election game, there is always a strong alliance stable state that is also a weak alliance stable state and a party stable state but hardly a global stable state. However, given an alliance structure, existence of a party stable state or a weak alliance stable state is yet to be concluded, for it has only been verified in a small subspace of the entire problem space.

Since there is always a strong alliance stable state (which is also a weak alliance stable state and a party stable state), each party might play its strategy in order to achieve such a stable state. However, there are often too many party stable states and weak alliance stable states, making parties unable to choose their strategies for sure. Also a strong alliance stable state is highly likely to correspond to a non-cooperating case in which parties play on their own, which lacks the specific cooperating ingredient in OLPR election games. Thus, the condition under which there is a strong alliance stable state given a cooperating alliance structure along with other possible interesting stable states should be studied in the future.