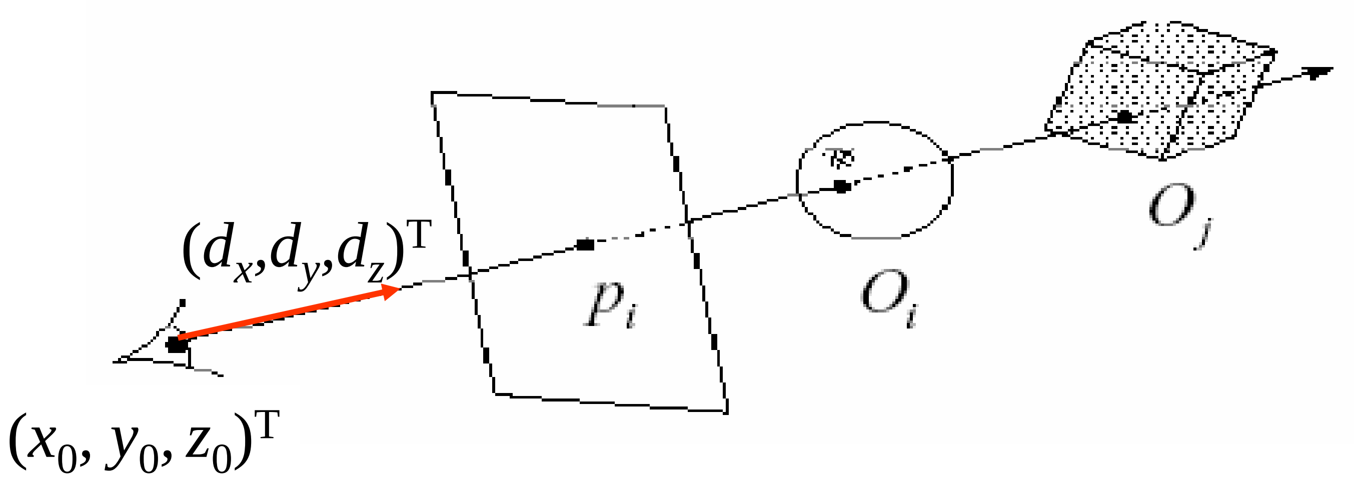
Lecture 11 Supplementary note:

Ray-Object Intersections

Mathematics

- The heart of any ray tracer, or ray casting for hidden surface removal, is the intersection routines.
- Each kind of primitive has different properties, so we have different intersection equations.



Parametric Ray Equation

- Let
  - the COP be $\mathbf{P}_0 = (x_0 , y_0 , z_0)^T$ and
  - the viewing direction be $\mathbf{D} = (d_x , d_y , d_z)^T$
- Any point P lying on the eye ray is given by:

$$\mathbf{P} = \mathbf{P}_0 + \mathbf{D}\, t$$

- Or writing each coordinate separately:

$$x = x_0 + d_x\, t$$
$$y = y_0 + d_y\, t$$
$$z = z_0 + d_z\, t$$

Ray Parameterization

- The parametric ray equation is given by:

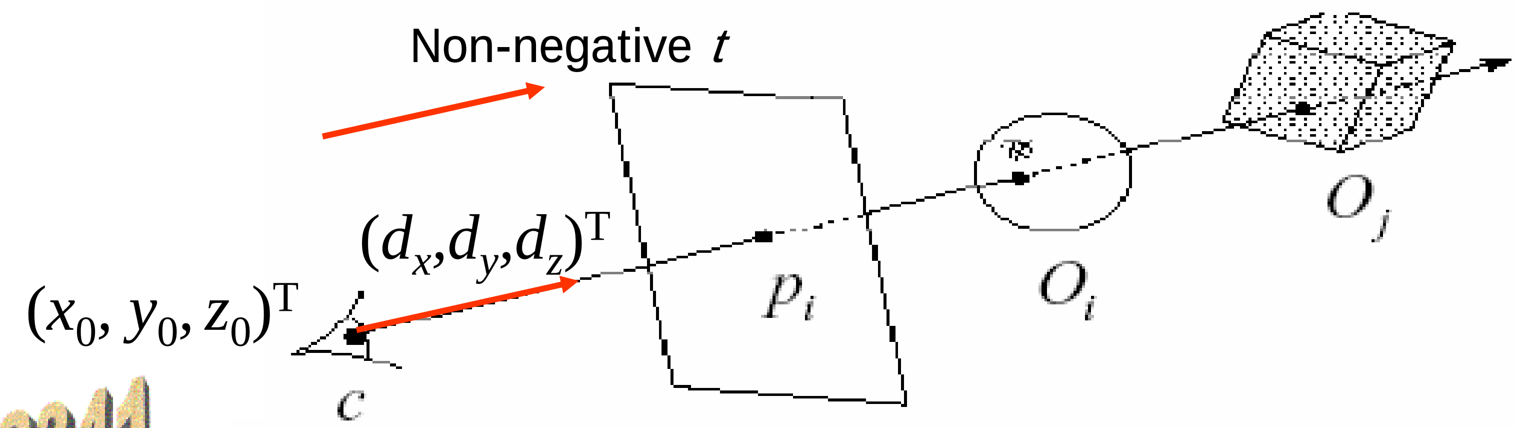
$$\mathbf{P} = \mathbf{P}_0 + \mathbf{D}\, t$$

- Points along the line of sight is parametrized by *t*:

  t = 0 , at COP (eye/viewpoint)

  t < 0 , behind COP

  t > 0 , in front of COP



Mathematics

- Given an implicit surface (i.e., spheres and other quadrics defined by an implicit equation)

$$F(x, y, z) = 0$$

- In the followings, all surface equations are assumed to be in the object space coordinate system. Therefore, we need to transform the ray before testing for intersection.

Intersecting Spheres

- The (implicit) equation of a unit sphere is given by:

$$x^2 + y^2 + z^2 = 1$$

- Assuming a unit sphere (radius is equal to one). Substituting the parametric ray equation yields the following:

$$(d_x x_0^2 + d_y y_0^2 + d_z z_0^2)\, t^2 + 2(d_x x_0 + d_y y_0 + d_z z_0)\, t + (x_0 + y_0 + z_0) - 1 = 0$$

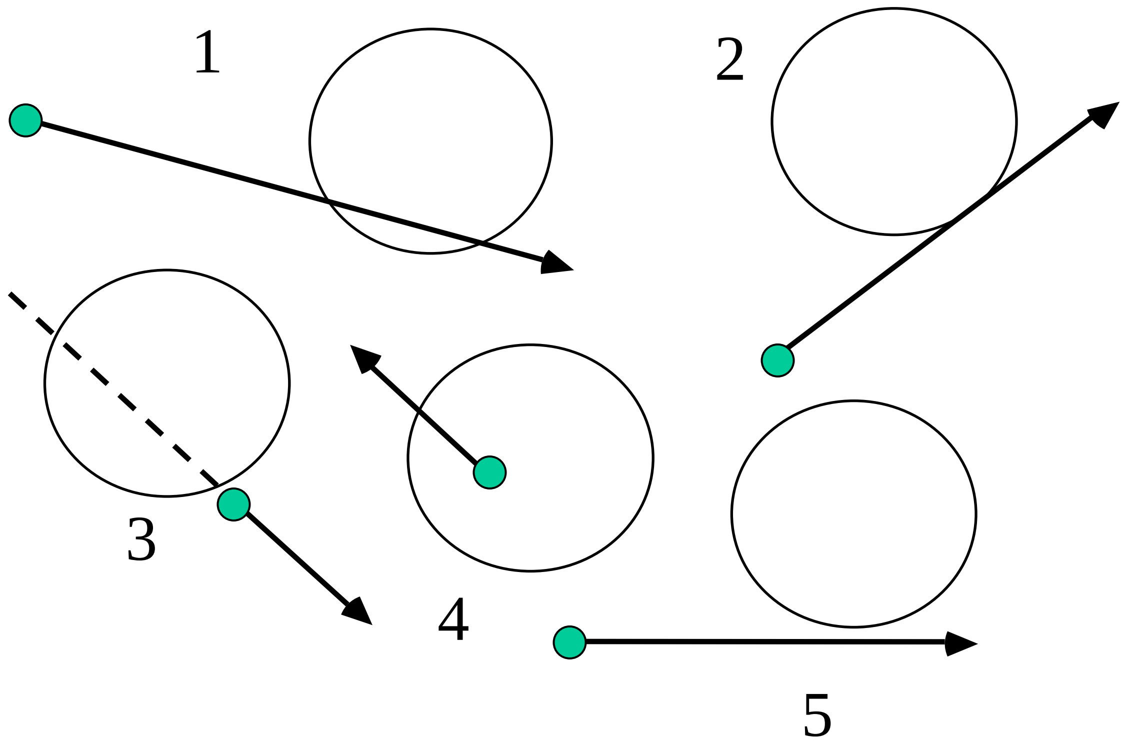
which is a quadratic equation in *t*.

Intersecting Spheres

- Solving the quadratic equation in t gives the solution.
- Ray misses the sphere if the discriminant is negative.
- If the discriminant is non-negative, the smallest positive t is taken.
- Else, the intersection point is given by:

$$x = x_0 + d_x \, t_1$$
$$y = y_0 + d_y \, t_1$$
$$z = z_0 + d_z \, t_1$$

Possible cases



1. Ray intersects sphere twice with t>0
2. Ray tangent to sphere
3. Ray intersects sphere with t<0
4. Ray originates inside sphere
5. Ray does not intersect sphere

Intersecting Quadrilaterals

- Solving a ray-plane equation determines if the ray hits the polygon plane. It is followed by an extent check to see if the ray hits the polygon.

- Let's write the ray equation as:

$$\mathbf{P} = \mathbf{P}_0 + \mathbf{D}\, t$$

which defines a ray as:

$$\mathbf{P}_0 = (x_0 , y_0 , z_0)^T$$
$$\mathbf{D} = (d_x , d_y , d_z)^T$$

Intersecting Quadrilaterals

- Define the plane in terms of [*A B C D*] as:

$$A\,\mathrm{x} + B\,\mathrm{y} + C\,\mathrm{z} + D = 0$$

- Note: the unit vector normal of the plane is defined by:

$$\boldsymbol{P}_{normal} = \boldsymbol{P}_n = [A\ B\ C\,]^{\mathrm{T}}$$

Intersecting Quadrilaterals

- Substituting the ray equation into the plane equation yields:

$$A(x_0 + d_x t) + B(y_0 + d_y t) + C(z_0 + d_z t) + D = 0$$

- Solving for t

$$t = \frac{-(Ax_0 + By_0 + Cz_0 + D)}{Ad_x + Bd_y + Cd_z}$$

- In vector form, the equation becomes

$$t = \frac{-(\vec{P}_n \cdot P_0 + D)}{\vec{P}_n \cdot \vec{D}}$$

- The vector equation will have no solution if the dot product of $\boldsymbol{P}_n$ and $\boldsymbol{D}$ is zero (ray direction exactly perpendicular to plane normal).

Intersecting Quadrilaterals

- Define

  $v_d = \mathbf{P_n} \cdot \mathbf{D}$

  $v_0 = -(\mathbf{P_n} \cdot \mathbf{P_0} + D)$

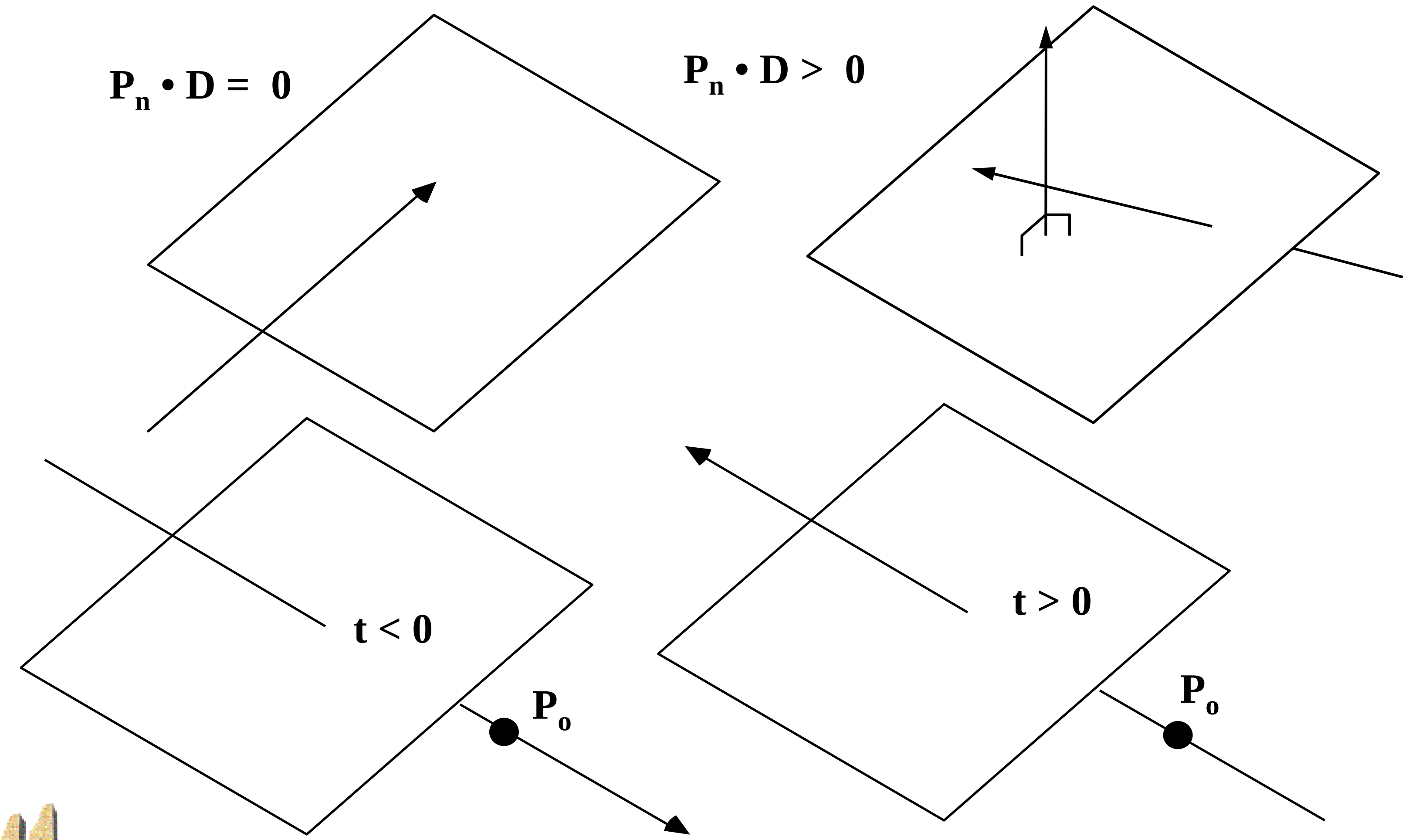
- Hence,

$$t = v_0 / v_d$$

- If t < 0, then the line defined by the ray intersects the plane behind the COP. Therefore, no intersection actually occurs.
- Else, the intersection point is given by:

$$\mathbf{P} = \mathbf{P}_0 + \mathbf{D}\,(v_0 / v_d)$$

Possible Cases



Intersecting Quadrilaterals

- Further extent check is required if the intersection point lies within the region bounded by the quadrilaterals

Any fast way to do this?

Intersecting a disk

- Intersecting circles is similar to intersecting quadrilaterals
- The extent check, after computing the intersection point, becomes one of using the circle equation

- Consider a circle lying on the z=0 plane. If the ray intersects the z=0 plane, it also intersects the circle if:

$$x^2 + y^2 - 1 \leq 0$$

Intersecting Cylinders

- Recall the parametric ray equation is:

$$x = x_0 + d_x t$$
$$y = y_0 + d_y t$$
$$z = z_0 + d_z t$$

- The equation for an infinite cylinder (along Z-axis) is:

$$x^2 + y^2 - 1 = 0$$

- Substituting the ray equation yields a quadratic equation in *t*:

$$(x_0 + d_x t)^2 + (y_0 + d_y t)^2 - 1 = 0$$
$$t^2(d_x^2 + d_y^2) + 2(x_0 d_x + y_0 d_y)t + (x_0^2 + y_0^2) - 1 = 0$$

- An extent check is applied for a finite cylinder.

Intersecting Cones

- The implicit equation for a cone is

$$x^2 + y^2 - z^2 = 0$$

- Substituting the ray equation into the above yields a quadratic equation in t:

$$(x_0 + d_x t)^2 + (y_0 + d_y t)^2 - (z_0 + d_z t)^2 = 0$$

$$t^2(d_x^2 + d_y^2 - d_z^2) + 2(x_0 d_x + y_0 d_y - z_0 d_z)t + (x_0^2 + y_0^2 - z_0^2) = 0$$

- Compute the discriminant, and solve for t if the discriminant is non-negative.