Principles of Programming Languages

COMP251: Functional Programming in SML

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Part I

Introduction

ML vs. Scheme/LISP

ML is another functional programming language that evolved from the LISP community, adding several important features.

- Prettier concrete syntax — ML programs support infix operators and statement separators like in C/C++, eliminating Scheme/LISP's heavy use of full parenthesization. However, this eliminates Scheme/LISP's advantage of naturally processing programs as data.
- Strong typing — type checking is a very helpful safety feature in software engineering, like in C/C++.
- Type inference — usually, you can omit the declaration of the type of a variable, because ML can automatically infer what the type should be from the context of the expression the variable occurs in. So the convenience and ease-of-use of ML is closer to Scheme/LISP where you never have to declare types of variables. (Standard C++ still doesn't support this!)
- Patterns — built-in pattern matching allows a declarative style of programming that can be very clean.

Standard ML

Standard ML or SML is a dialect of ML that resulted from an international community standardization process. Consequently, there are many implementations of SML.

A few other dialects still exist. Of particular note, CAML and its object-oriented extension O'Caml are very efficient implementations rivalling C++, that have a significant following.

SML (Standard Meta Language)

SML (/usr/local/sml/bin/sml) supports:

- Higher-Order Functions: composite functions in math. e.g. $f(g(h(x)))$
- Abstract Data Types: type = data + functions (as in OOP).
- Polymorphism: functions, data types (c.f. template in C++).
- Strong Typing: Every expression has a type which can be determined at compile time. (c.f. C++ is not. e.g. virtual function)
- Static Scope: All identifier references resolved at compile time.
- Rule-Based Programming: Actions are selected through if-then pattern-matching rules (c.f. AI language like Prolog).
- Type-Safe Exception Mechanism: to handle unusual situations arising at run-time. e.g. division-by-zero.
- Modules: an ML program = a set of interdependent modules glued together using functors.

Part II

Types, Values, Patterns

5 Basic Types, 3 Composite Types

TYPE	SYMBOL	EXAMPLE	OPERATIONS/TYPES
unit	()	()	—
boolean	bool	true, false	not, andalso, orelse
integer	int	2, 0, 87	$\sim$, $+$, -, $*$, div, mod
real	real	1.3, 3E2	$\sim$, $+$, -, $*$, $/$
string	string	"hello"	^
tuple	(...)	(1, "yes", 2.5)	int*string*real
list	[...]	[3, 8, 1, 9]	int list
record	{...}	{name="Eagle", dead=true}	{name:string,dead:bool}

Basic Types

- **unit** is similar to **void** in C. It is used
  - whenever an expression has no interesting value.
  - when a function is to have no arguments.
- Negative **int** or **real** values are denoted using the unary operator $\sim$ instead of the usual minus sign.
- Integer division uses **div** and **mod**, and real number division uses /.
- **NO** implicit coercion!
- The boolean operators **andalso** and **orelse** perform short-circuit evaluations: i.e.
  E_1 **andalso** E_2 $\Rightarrow$ will NOT evaluate E_2 if E_1 is **false**.
  E_1 **orelse** E_2 $\Rightarrow$ will NOT evaluate E_2 if E_1 is **true**.

Example: int/real

```
- ();
val it = () : unit

- 5 + 13;
val it = 18 : int

- ~5 + 13;
val it = 8 : int

- floor(123.6);
val it = 123 : int

- floor(~123.6);
val it = ~124 : int
```

Example: String

```
- "Hong"^" "^"Kong";
val it = "Hong Kong" : string

- size "Hong Kong";
val it = 9 : int

- size "Hong"^" "^"Kong";
stdIn:69.1-69.23 Error:
  operator and operand don't agree [tycon mismatch]
  operator domain: string * string
  operand:         int * string
  in expression:   size "Hong" ^ " "

- size("Hong"^" "^"Kong");
val it = 9 : int
```

Example: Type Checking

```
- 5/6;
stdIn:50.2 Error: overloaded variable not defined at type
  symbol: /
  type: int

- real(5)/6;
stdIn:1.1-49.6 Error: operator and operand don't agree
  operator domain: real * real
  operand:         real * int
  in expression:
    real 5 / 6

- real(5)/real(6);
val it = 0.833333333333 : real
```

Example: Boolean Expression

```
- if 2=3 then "don't worry" else "be happy";
val it = "be happy" : string

- if "don't worry"="be happy" then 1 else 2;
val it = 2 : int

- if 2=3 then "don't worry" else 4;
stdIn:1.1-61.3 Error: types of rules don't agree [literal]
  earlier rule(s): bool -> string
  this rule:       bool -> int
  in rule:         false => 4
```

- "**if**<bool-exp> **then**<then-exp> **else**<else-exp>" always come together; and its value is that of <then-exp> if <bool-exp> is true, otherwise that of <else-exp>.
- <then-exp> and <else-exp> must match in their types.

Composite Type: Tuple

```
- (4, true, "cat");
val it = (4,true,"cat") : int * bool * string
- (if 3=8 then "X" else "Y", 9.5/0.5, 5 div 2);
val it = ("Y",19.0,2) : string * real * int
- (14 mod 3, not false) = (1+1, true);
val it = true : bool

- #2("for", "your", "info");
val it = "your" : string
```

- Ordered n-tuple: $(e_1, e_2, \ldots, e_n)$.
- Like vector in Scheme.
- The n expressions may be of mixed types.
- 2 n-tuples are equal if their corresponding components are equal.
- "#" is the item selection operator.

List

- Empty list: **nil** or [ ];
- **nil** : 'a list ⇒ a polymorphic object.
- $[e_1, e_2, \ldots, e_n]$ is an abbreviation for e_1::e_2::. . .::e_n::**nil**.
- :: is the list constructor pronounced as "cons".
- :: is an infix operator which is right associative.
- <new-list> = <item>::<list>.

```
1::2::3::nil = 1::(2::(3::nil))
             = 1::(2::[3])
             = 1::[2,3]
             = [1,2,3]
```

- Equality on 2 lists is item-by-item.

List Operators

- cons operator: **::** : ‘a item $*$ ‘a list $\longrightarrow$ ‘a list
- head operator: **hd()** : ‘a list $\longrightarrow$ ‘a item
- tail operator: **tl()** : ‘a list $\longrightarrow$ ‘a list
- append operator: **@** : ‘a list $*$ ‘a list $\longrightarrow$ ‘a list

List Examples

```
- hd([1,2,3,4]);
val it = 1 : int

- tl([1,2,3,4]);
val it = [2,3,4] : int list

- hd([1,2,3,4])::tl([1,2,3,4]);
val it = [1,2,3,4] : int list

- [5,6]@tl([1,2,3,4]);
val it = [5,6,2,3,4] : int list
```

Record

- c.f. **struct** in C.
- Syntax: $\{\ label_1 = E_1,\ label_2 = E_2,\ \ldots\ \}$
- Order does NOT matter since the fields are labelled.
- Tuples are actually short-hands for records.
  $(E_1,\ E_2,\ E_3) = \{\ 1{=}E_1,\ 2{=}E_2,\ 3{=}E_3\ \}$

```
- {name="bird", age=5, dead=true};
val it = {age=5,dead=true,name="bird"}
         : {age:int, dead:bool, name:string}
```

```
- {name="bird", age=5, dead=true}
   = {age=5, dead=true,name="bird"};
val it = true : bool
```

Identifiers

BNF for alphanumeric identifiers:

```
<Id>           ::= <First_Char><Other_Chars>
<First_Char>   ::= [A-Z]|[a-z]|'
<Other_Chars>  ::= <empty>|<Other_Char><Other_Chars>
<Other_Char>   ::= [A-Z]|[a-z]|[0-9]|['_]
```

BNF for symbolic identifiers:

```
<Id>          ::= <S_Char>|<S_Char><Id>
<S_Char>      ::= [+-/*<>=!@#%^`~\$?:]
```

- Disallow mixing the 20 symbols with alphanumeric characters.
- '<Other_Char> are *alpha* variables ONLY used for data types.
- Symbolic identifiers should be used for user-defined operators.

Identifiers: Value Binding

Syntax: **val** < *identifier* > **=** < *expression* >**;**

```
- val a_df = 3+2;  (* c.f. const int a_df = 3+2; in C++ *)
val a_df = 5 : int

- val a'a = "Albert"^" "^"Einstein";
val a'a = "Albert Einstein" : string

- val a1b2 = 2;
val a1b2 = 2 : int

- val +++$$$ = 9*3; (* may hold integral value *)
val +++$$$ = 27 : int

- +++$$$ + +++$$$;  (* Though you don't want to do that *)
val it = 54 : int
```

Pattern Matching

- Pattern matching with tuples

```
- val (left, right) = ("Einstein", 4);
val left = "Einstein" : string
val right = 4 : int
```

- Pattern matching with lists

```
- val x::y = [5,6,7,8];   (* [5,6,7,8] = 5::[6,7,8] *)
val x = 5 : int
val y = [6,7,8] : int list
```

- Pattern matching with records

```
- val {flag=y,count=x} = {count=2,flag=true};
val x = 2 : int
val y = true : bool
```

Pattern Matching: Wildcard Pattern

The wildcard pattern "_" (underscore symbol) may be used for terms that you don't care in pattern matching.

```
-  val (left,_) = ("Einstein", 4);
val left = "Einstein" : string

- val _::a = [1,2,3];
val a = [2,3] : int list

- val x::_::z = [[1,2],[3,4],[7,9],[0,0]];
val x = [1,2] : int list
val z = [[7,9],[0,0]] : int list list
```

Pattern Matching: Bug

Identifiers cannot duplicate in various parts of a pattern.

```
- val (x, x::y) = (3, [3,4,5]);
stdIn:1.1-287.4 Error: duplicate variable in pattern(s): x

- val (x, x) = (3,3);
stdIn:1.1-279.7 Error: duplicate variable in pattern(s): x
```

Part III

SML Functions

Functions: It is "fun"

- Syntax: **fun** <identifier> (<parameter-list>) = <expression>;
- Parameter passing method: Call-By-Value.

```
- fun square(x) = x*x;
val square = fn : int -> int

- fun square x = x*x; (* parentheses are optional *)
val square = fn : int -> int

- square 4;
val it = 16 : int

- fun first (x,y) = x;    first (3, "foo");
val first = fn : 'a * 'b -> 'a
val it = 3 : int
```

Type of Functions

Each identifier, variable or function, has a type.
Function : <domain type> → <range type>

- Argument type may be explicitly specified with :< *type* >.
  e.g. A function whose input is a **real** number and which returns a **real** number:

```
- fun f_square(x: real) = x*x;
val f_square = fn : real -> real


- fun f_square(x):real = x*x;     (* Another way *)
```

- A function whose domain type is a tuple ('a type, 'b type) and whose range type is 'a.

```
- fun first (x,y) = x;
val first = fn : 'a * 'b -> 'a
```

More Complex Functions

- Defined with boolean expressions.

```
- fun greater(x,y) = if x > y then x else y;
```

```
- fun factorial x = if x = 0
=     then 1    (* Initial ‘‘=’’ is continuation symbol *)
=     else x*factorial(x-1);
```

- Defined by enumerating **ALL** cases with pattern matching ($\Rightarrow$ more readable).

```
- fun factorial 0 = 1
    | factorial x = x * factorial(x-1);
```

Functions: Bug

When functions are defined by case analysis, SML issues a warning or an error if

- Not all cases are covered.

```
- fun myhead(head::tail) = head;
stdIn:266.1-266.30 Warning: match nonexhaustive
          head :: tail => ...
val myhead = fn : 'a list -> 'a
```

- A case is redundant because of earlier cases.

```
- fun nonsense(_) = 3 | nonsense(0) = 5;
stdIn:275.1-275.47 Error: match redundant
          _ => ...
    -->   0 => ...
```

Type System & Inference

- Type System: for a language is a set of rules for associating a type with an expression in the language.
- Type Inference: to deduce the type of an expression
- Basic Rule: if $f : A \longrightarrow B$, and a has type A then $f(a)$ must be of type B.

Whenever possible, ML automatically infers the type of an expression.

Type Inference Rules

1. Types of operands and results of arithmetic operators must agree.

$$4.8/(a - b)$$

2. Types of operands of comparison operators must agree.

$$x = 1; \qquad\qquad x < y$$

3. For the **if-then-else** expression (not statement!), **then**-expression and **else**-expression must be of the same type.

$$\text{if} \quad x > 1 \quad \text{then} \quad y \quad \text{else} \quad z$$

4. Types of actual and formal parameters of a function must agree.

$$\text{fun } g(x) = 2 * x; \qquad g(5);$$

Higher-Order Functions (I)

Functions taking functions as arguments:

```
- fun square x = x*x; fun twice x = 2*x;

- fun apply5 f = f 5; apply5 square;
val apply5 = fn : (int -> 'a) -> 'a
val it = 25 : int

- fun apply f x = f(twice(x)); apply square 3;
val apply = fn : (int -> 'a) -> int -> 'a
val it = 36 : int

- fun first x y = x; first 2 "foo";
val first = fn : 'a -> 'b -> 'a
val it = 2 : int
```

Higher-Order Functions (I) ..

- Function application is left-associative.

  Thus, (first x y) = ((first x) y).

- Operator $\rightarrow$ is right-associative.

  Thus, $'a \rightarrow 'b \rightarrow 'a = 'a \rightarrow ('b \rightarrow 'a)$.

- i.e. first() has domain type = 'a, range = $'b \rightarrow 'a$.

- i.e. first() takes an 'a value and returns another function which takes a 'b value and returns an 'a value.

Functions returning function:

```
- fun sq_or_twice  x = if x > 0 then square else twice;
val sq_or_twice = fn : int -> int -> int

- (sq_or_twice 2) 5;
val it = 25 : int

- sq_or_twice 2;
val it = fn : int -> int
```

Type Inference: Example 1

fun H f x = f x

$$\left\{ \begin{array}{l} H\ f = g \\ g\ x = y \\ f\ x = y \end{array} \right. \Rightarrow \left\{ \begin{array}{l} type(H) = type(f) \rightarrow type(g) \\ type(g) = type(x) \rightarrow type(y) \\ type(f) = type(x) \rightarrow type(y) \end{array} \right.$$

Let $type(x) = {}'a$ and $type(y) = {}'b$, then

$$\left\{ \begin{array}{l} type(g) = type(f) = {}'a \rightarrow {}'b \\ type(H) = ('a \rightarrow {}'b) \rightarrow ('a \rightarrow {}'b) \end{array} \right.$$

Type Inference: Example 2

fun H f x = G(f x) where type(G) = 'a $\rightarrow$ 'b.

$$\left\{\begin{array}{c} H\ f = g \\ g\ x = y \\ f\ x = z \\ G\ z = y \end{array}\right. \Rightarrow \left\{\begin{array}{l} type(H) = type(f) \rightarrow type(g) \\ type(g) = type(x) \rightarrow type(y) \\ type(f) = type(x) \rightarrow type(z) \\ type(G) = type(z) \rightarrow type(y) \equiv {}'a \rightarrow {}'b \end{array}\right.$$

Let $type(x) = {}'c$, then

$$\left\{\begin{array}{l} type(f) = {}'c \rightarrow {}'a \\ type(g) = {}'c \rightarrow {}'b \\ type(H) = ({}'c \rightarrow {}'a) \rightarrow ({}'c \rightarrow {}'b) \end{array}\right.$$

Functions on List: Examples

- In general, a function on list must deal with the 2 cases:
  - [ ] or nil
  - head::tail

```
- fun len([]) = 0  |  len(x::tail) = 1 + len(tail);

- fun sum([]) = 0  |  sum(x::tail) = x + sum(tail);

- fun mean L = sum L div len L;

- mean [1,2,3];
val it = 2 : int

- fun append([], L2) = L2
    | append(x::tail, L2) = x::append(tail, L2);

- append([3,5], [9,8,7]);
val it = [3,5,9,8,7] : int list
```

List Function: map

- The built-in map() has 2 arguments: a function f() and a list.
- It applies function f() to each element of the list.

$$
\begin{array}{l}
\textbf{fun}\ \mathrm{map}\ f\,[\,] = [\,] \\
\quad |\quad \mathrm{map}\ f\,(\mathrm{head::tail}) = (f\,\mathrm{head})::(\mathrm{map}\ f\,\mathrm{tail});
\end{array}
$$

- Type of list: 'a list
- Type of f: 'a $\rightarrow$ 'b
- Type of map: ('a $\rightarrow$ 'b) $\rightarrow$ 'a list $\rightarrow$ 'b list

map: Examples

```
- fun odd x = (x mod 2) = 1;
val odd = fn : int -> bool

- map odd [1,2,3];
val it = [true,false,true] : bool list

- map odd;
val it = fn : int list -> bool list

- map;
val it = fn : ('a -> 'b) -> 'a list -> 'b list
```

List Function: filter

- filter applies a boolean test function to each element of a list, removing the element should the test fail.

fun filter f [ ] = [ ]
  | filter f (head::tail) = **if** (f head)
    then head::(filter f tail)
    else (filter f tail);

```
- filter odd [1,2,3,4,5];
val it = [1,3,5] : int list

- filter;
val it = fn : ('a -> bool) -> 'a list -> 'a list

- filter odd;
val it = fn : int list -> int list
```

List Function: reduce

- reduce accumulates a result from a list.

fun reduce f [ ] v = v
| reduce f (head::tail) v = f (head, reduce f tail v);

```
- reduce add [1,2,3,4,5] 0;
val it = 15 : int

- reduce;
val it = fn : ('a * 'b -> 'b) -> 'a list -> 'b -> 'b

- reduce add;
val it = fn : int list -> int -> int

- reduce add [1,2,3,4,5];
val it = fn : int -> int
```

List Function: Example

```
- fun reverse_([], L2) = L2
    | reverse_(x::tail, L2) = reverse_(tail, x::L2);
- fun reverse L = reverse_(L, []);

- reverse ["D","O","G"];
val it = ["G","O","D"] : string list
```

- rev: 'a list → 'a list, is SML's built-in operator to do that.

```
- rev ["D","O","G"];
val it = ["G","O","D"] : string list
```

Anonymous Functions

Syntax: **fn** <formal parameter> $\Rightarrow$ <body>

- An *anonymous function* is a function without a name.
- Used when only a locally defined function is needed.

```
- map (fn x => x*x) [2,3,4];
val it = [4,9,16] : int list
```

```
- map (fn (x,_) => x) [(1,2), (3,4), (5,6)];
val it = [1,3,5] : int list
```

Functions as Values

Functions are the first-class objects in SML, they can be input as arguments, returned as return-values, and also created as values.

```
- val square = fn x => x*x; square 4;
val square = fn : int -> int
val it = 16 : int

- val f = square; f 4;
val f = fn : int -> int
val it = 16 : int

- val g = map square;
val g = fn : int list -> int list

- g [1,2,3,4];
val it = [1,4,9,16] : int list
```

Composite Functions

Given: f: 'b $\rightarrow$ 'c and g: 'a $\rightarrow$ 'b .

Define a new function: $h(x) = f \circ g(x) \equiv f(g(x))$: 'a $\rightarrow$ 'c.

i.e first apply function g() to an input x of 'a type, returning a value of 'b type, which is then piped into function f() to give the final result of 'c type.

```
- fun square x = x*x;       fun twice x = 2*x;
val square = fn : int -> int
val twice = fn : int -> int


- val sq_twice = square o twice; (* Use val NOT fun *)
val sq_twice = fn : int -> int


- sq_twice 3;
val it = 36 : int
```

Eager (Innermost) Evaluation

```
fun f(x) = if x = 1 then 1 else x*f(x-1);

f(1+1) = f(2)
       = if 2 = 1 then 1 else 2 * f(2-1)
       = 2 * f(1)
       = 2 * { if 1 = 1 then 1 else 1 * f(1-1) }
       = 2 * 1
       = 2
```

Actual parameters are evaluated before they are passed to functions $\Rightarrow$ Call-By-Value.

Lazy (Outermost) Evaluation

```
f(1+1) = if (1+1) = 1 then 1 else (1+1) * f((1+1)-1)
       = if 2 = 1 then 1 else (1+1) * f((1+1)-1)
       = (1+1) * f((1+1)-1)
       = 2 * f((1+1)-1)
       = 2 * { if ((1+1)-1) = 1 then 1 else
                 ((1+1)-1) * f(((1+1)-1) - 1) }
       = 2 * 1
       = 2
```

Actual parameters are evaluated only when they are needed.

Eager vs. Lazy Evaluation

- Give same result if the execution terminates.
- But consider the following 2 examples:

```
if X = 0 or Y/X > 5 then ... else ...;
```

```
X + (Y == 0 ? 2 : 4/Y);
```

Expression Evaluation in ML

- For function application:

  eager evaluation of actual parameters

- For boolean expression:

  short-circuit evaluation = lazy evaluation

- E_1 **or** E_2 actually is a function $\textbf{or}(E_1, E_2)$. ML's eager evaluation of actual parameters may not give the same result as required by short-circuit evaluation.

  $\Rightarrow$ a new operator **orelse**. (same for **andalso**)

Creating New Infix Operators

Left-associative: **infix** <precedence-level> <operator id>.
Right-associative: **infixr** <precedence-level> <operator id>.

- If omitted, <precedence-level> = 0 — the min. level.
- The highest precedence level is 9 in our SML.

PRECEDENCE	OPERATORS	ASSOCIATIVITY	COMMENTS
3	o	—	function composition
	:=	—	assignment
4	$=, <>, <, >, \leq, \geq$	left	relational operators
5	::	right	list constructor
	@	right	list concatenation
6	+, -	left	add/subtract
	^	left	string concatenation
7	$*$, /, div, mod	left	multiply/divide

New Operator ..

```
(* First create the function *)
- fun **(a,0) = 1 | **(a,b) = a * **(a,b-1);
val ** = fn : int * int -> int

- **(2,5);
val it = 32 : int

- infix 7 **;     (* Make ** left-associative *)
infix 7 **

- 4 + 2**5 - 6;      2**3**2;
val it = 30 : int
val it = 64 : int

- infixr 7 **;     (* Make ** right-associative *)
infixr 7 **

- 2**3**2;
val it = 512 : int
```

Operators as Functions

- Functions are called as prefix operators.
- Built-in operators like +, −, ∗, / are called as infix operators.
- Internally, infix operators are actually functions. To use them as functions: **op** operator-symbol.

```
- op+(2,3); op*(4,5);
val it = 5 : int
val it = 20 : int

- reduce op+ [2,3,4] 0; reduce op*  [2,3,4] 1;
val it = 9 : int
val it = 24 : int

- op+;
val it = fn : int * int -> int
```

Part IV

Static Scope: let Expression

let Expression

let

  val <1st-identifier> $= <E_1>$;

  val <2nd-identifier> $= <E_2>$;

  ...

in

  <expression>

end

- The semicolons at the end of each **val** statements is optional.
- c.f. Declaration of local variables in C++

let: val Example

```
- val z =
      let
        val x = 3;
        val y = 5;
      in
        x*x + 3*y
      end;
val z = 24 : int
```

- As spaces are immaterial, the statement may as well be written all in one single line as follows:

```
val z = let val x = 3 val y = 5 in x*x + 3*y end;
```

- To avoid too many **val** statements in the **let**-part, one may use tuples to group all identifiers as follows:

```
val z = let val (x, y) = (3, 5) in x*x + 3*y end;
```

Nested let Example

```
- let val x = 3.0  val y = 5.0 in
      let val a = x+y  val b = x-y in
          let val f = a*b*x  val g = a/b/y  in  f/g  end
      end
end;
```

Quiz: What is the output?

let: fun Example

- Let's rewrite the function reverse() with a locally defined function, rev_().

```
fun reverse L =
    let fun rev_([], L2) = L2
          | rev_(x::tail, L2) = rev_(tail, x::L2)
    in  rev_(L, []) end;
```

- Identifiers with the same names are resolved using the static lexical scope rule.

```
fun weird(x: real) =
    let val x = x*x
        val x = x*x
    in  x*x*x end;
- weird 2.0;    (* What is the result? *)
```

Part V

New Datatypes

Defining New Datatypes

Syntax: **datatype** <type-name>
$= <$1st-constructor$> \mid <$2nd-constructor$> \mid \ldots$

- A simple example:

```
datatype Primary_Lights = red | green | blue;

- red;
val it = red : Primary_Lights
```

- c.f. enumeration in C++
  enum Primary_Lights = { red, green, blue };

Constructors of Datatype

- More complex objects can be constructed too. e.g.

```
datatype Money = nomoney  fun amount nomoney = 0
   | coin of int              | amount(coin(x)) = x
   | note10 of int            | amount(note10(x)) = 10*x
   | note100 of int           | amount(note100(x)) = 100*x
   | check of string*int;     | amount(check(bank,x)) = x;

- amount (note100(2));
val it = 200 : int
```

- Money has 5 constructors: nomoney as a constant constructor, coin(int), note10(int), note100(int), and check(string, int).
- Any function on Money should deal with have 5 cases, one for each constructor.

Recursive Datatype: Differentiation Example

```
- datatype expr = constant of int
                | variable of string
                | sum of expr * expr
                | product of expr * expr;
- val zero = constant 0; val one = constant 1;

- fun D x (constant _) = zero
    | D x (variable z) = if x = z then one else zero
    | D x (sum(e1, e2)) = sum(D x e1, D x e2)
    | D x (product(e1, e2)) =
          let val term1 = product(D x e1, e2)
              val term2 = product(e1, D x e2)
          in sum(term1, term2) end;
val D = fn : string -> expr -> expr
```

Recursive Datatype: Differentiation Example ..

- **expr** has 4 constructors: constant(int), variable(string), sum(expr, expr), product(expr, expr).
- Declarations of "zero" and "one" is necessary in order to have an output type of **expr**; you can't use integers 0 and 1.

In order to use the new datatype **expr** and the differentiation function **D**, one has to convert a mathematical expression to **expr**. For example, to differentiate "`x*x + 5*x`":

Recursive Datatype: Differentiation Example ...

```
- Compiler.Control.Print.printDepth := 10;
val it = () : unit

- val term = sum(product(variable "x", variable "x"),
                 product(constant 5, variable "x"));
val it =
  sum (product (variable "x",variable "x"),
       product (constant 5,variable "x")) : expr

- D "x" term;
val it = sum (sum (product (constant 1,variable "x"),
                   product (variable "x",constant 1)),
              sum (product (constant 0,variable "x"),
                   product (constant 5,constant 1))) : expr
```

Polymorphic Datatype: Binary Tree Example

```
datatype 'a tree =
    empty_tree | leaf of 'a | node of 'a tree*'a tree;
```

- The **'a tree** has 3 constructors: empty_tree (constant constructor), leaf('a tree), and node('a tree, 'a tree).

```
- fun leafcount(empty_tree) = 0
    | leafcount(leaf(x)) = 1
    | leafcount(node(L,R)) = leafcount(L) + leafcount(R);
val leafcount = fn : 'a tree -> int

- val x = node(node(leaf(1), leaf(2)), leaf(3));
val x = node (node (leaf #,leaf #),leaf 3) : int tree

- leafcount x;
val it = 3 : int
```

Abstract Data Types

```
abstype 'a stack = stack of 'a list
    with
        val emptystack = stack [];
        fun SK_empty(stack y) = y = nil;
        fun SK_push(x, stack y) = stack (x::y);
        fun SK_pop(stack y) = (hd(y), stack(tl(y)));
        fun SK_list(stack y) = y;
    end;

val x = emptystack;
val y = SK_push(3, SK_push(4,x));
val z = SK_pop y;
SK_list x; SK_list y; SK_list (#2(z));
SK_pop(#2(SK_pop(#2(SK_pop y))));
```

Part VI

Misc: Value Binding, Exception

Impure FP: Ref-Variables, Assignments

- Reference variable points to a value (c.f. indirect addressing):

  val <identifier> = **ref** <expression>.
- Assignment: <identifier> **:=** <expression>
- Dereference: **!**<identifier>

```
- val x = ref(2+3);              |  - val y = ref 9;
val x = ref 5 : int ref          |  val y = ref 9 : int ref
                                 |
- x := 9;                        |  - !x = !y;
val it = () : unit               |  val it = true : bool
                                 |
- x;                             |  - x = y;
val it = ref 9 : int ref         |   val it = false : bool

- !x;
val it = 9 : int
```

Value Binding and Environment

- The phrase: "**val** x = 17" is called a value binding; the variable x is bound to the value 17.
- When an identifier is declared by a value binding, a new identifier is "created" — it has nothing whatever to do with any previously declared identifier of the same name.
- Once an identifier is bound to a value, there is no way to change that value.
- Environment: the current set of ordered pairs (identifier, value) that are visible.

Environment: Example

- val x = 17;

val x = 17 : int

- val y = x;

val y = 17 : int

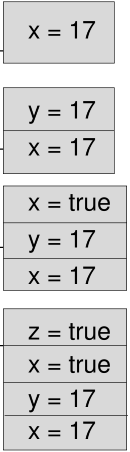
- val x = true;

val x = true : bool

- val z = x;

val z = true : bool

env:



Assignment and Side Effects

- val x = ref 0;

val x = ref 0 : int ref

state: { (x, 0) }

- x := 17;

val it = () : unit

state: { (x, 17) }

- val y = x;

val y = ref 17 : int ref

state: { (x, 17), (y, 17) }

- x := 9;

val it = () : unit

state: { (x, 9), (y, 9) }

- val z = x;

val z = ref 9 : int ref

state: { (x, 9), (y, 9), (z, 9) }

- The assignment x := 9 produces the side-effect such that not only x's derefenced value is changed, but also y's.

Alias and Side Effects

Alias: When a data object is visible through more than one name in a single referencing environment, each name is termed an alias.

- Examples: passed parameters by reference in a function, several pointers to the same object.
- Pitfall: programs are harder to understand.

Side Effects: An operation has side effects if it makes changes which persist after it returns.

- Examples : A function changes its parameters or modifies global variables (through assignments); printouts.
- Pitfall : programs are harder to understand, evaluation order of expressions becomes important.

Alias and Side Effects: Example

```
int x = 2, y = 5;
int Bad(int m) { return x+=m; }

void Swap(int* a, int* b)
{
    int temp = *a; *a = *b; *b = temp;
    x = 4;
}

int main()
{
    int* z = &x;
    int k = x * Bad(7) + x;

    printf("k = %d\n", k);
    Swap(&x, &y);
    printf("(x,y) = (%d,%d)\n", x, y);
}
```

Assignment and Value Binding: Example

```
val x = ref 0;

fun F y =
    let
        val w = 5;
    in
        y + 3*w + !x
    end;

F 1;
x := 10;
F 1;
val x = 999;
F 1;
val x = ref 999;
F 1;
```

- What are the values after each "F 1;" expressions?

Exception

Keywords: **exception, raise, handle** $=>$

```
- 8 + 9 div 0;
uncaught exception divide by zero

- exception DBZ;
exception DBZ

- fun //(a,b) = if b = 0 then raise DBZ else a div b;
val // = fn : int * int -> int

- infix 7 //;
infix 7 //
```

Exception ..

```
- fun g (x,y,z) = x + y // z;
val g = fn : int * int * int -> int
- g(8,9,3);
val it = 11 : int

- g(8,9,0);
uncaught exception DBZ
  raised at: stdIn:30.3-30.6

- fun f(x,y,z) = x + y // z handle DBZ => ~999;
val f = fn : int * int * int -> int

- f(8,9,0);
val it = ~999 : int
```

Load SML Programs, Libraries

- To load an SML program file which may contain value and function definitions, use "use"

```
- use "full-filename";
```

- To load lib, use "open"

```
- open Int;
- open Real;
- open Math;
- open String;
- open List;
- open IO;
- open TextIO;
```

Part VII

Summary

Summary

- $\surd$ A task is achieved through applications of functions.
- $\surd$ No pointers!
- $\surd$ No coercion!
- $\surd$ No side-effects!
- $\surd$ Assignment is replaced by value binding.
- $\surd$ Implicit type inference.
- $\surd$ Implicit memory management: Objects are allocated as needed, and deallocated when they become inaccessible.
- $\surd$ Pattern matching $\Rightarrow$ program by examples.
- $\surd$ Allow recursive definition of polymorphic datatypes.
- $\surd$ Simple exception handling.

Summary: FP vs. IP

IP:	Since IP languages are based on the von Neumann architecture, programmers must deal with the management of variables, assignment of values to them, memory locations, and sometimes even memory allocations. • Adv: efficient computation • Disadv: laborious construction of programs
FP:	Do not manipulate memory directly; no variables, no assignments. Instead they work on values that are independent of an underlying machine. • Adv: compact language, simple syntax, higher level of programming • Disadv: efficiency is sacrificed

Summary: FP vs. IP ..

IP:	Due to aliases and side effects, the effects of a subprogram or a block cannot be determined in isolation from the entire program.
FP:	Since they only manipulate values, there are no aliases nor side effects.
IP:	Explicit memory management.
FP:	Storage is allocated as necessary; and storage that becomes inaccessible is automatically deallocated and reclaimed during garbage collection.
IP:	The power comes from mimicking operations on the underlying computer architecture with assignments, loops, and jumps.
FP:	The power comes from recursion and treating functions as "first-class" values.