# Error Modeling and Estimation Fusion for Indoor Localization

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***Abstract*—There has been much interest in offering multimedia location-based service (LBS) to indoor users (e.g., sending video/audio streams according to user locations). Offering good LBS largely depends on accurate indoor localization of mobile stations (MSs). To achieve that, in this paper we first model and analyze the error characteristics of important indoor localization schemes, using Radio Frequency IDentification (RFID) and Wi-Fi. Our models are simple to use, capturing important system parameters and measurement noises, and quantifying how they affect the accuracies of the localization.**

**Given that there have been many indoor localization techniques deployed, an MS may receive simultaneously multiple co-existing estimations on its location. Equipped with the understanding of location errors, we then investigate how to optimally combine, or *fuse*, all the co-existing estimations of an MS's location. We present computationally-efficient closed-form expressions to fuse the outputs of the estimators. Simulation and experimental results show that our fusion technique achieves higher location accuracy in spite of location errors in the estimators.**

***Keywords*-Indoor localization, mobile users, multimedia location-based service**

## I. Introduction

With the advancement of mobile device capabilities and penetration of wireless access networks, many new types of mobile multimedia services become viable. Among those services, offering multimedia location-based service (LBS) is one with high growth and commercial potential. We give some examples below. 1) In large shopping malls, retailers may want to push their promotion ads (videos, text and/or images) to customers depending on the customer locations. 2) Standing in front of some masterpiece in a museum, a tourist may want to request video instruction from her mobile phones. 3) If we are able to track the location of a mobile phone, many multimedia arts can be designed. For instance, users can draw a picture or pattern based on their locations. LBS relies largely on accurate localization of mobile stations (MSs). Though Global Positioning System (GPS) has already achieved high accuracy, it only works well in outdoor open environment. Accurate indoor localization still remains challenging.

Many indoor localization techniques have been proposed, studied or deployed. Apart from those based on angle of arrival (AOA), time of arrival (TOA), time difference of arrival (TDOA) and received signal strength (RSS), two important ones make use of RFID and Wi-Fi. There are many RFID localization schemes, one of which is LANDMARC [1]. There are two components in LANDMARC: one is active tag, which is used as reference or target, the other is radio frequency reader. Locations of references near a target is used to estimate the target's position. However, there will be some error in the measurement of signal strength.

Using Wi-Fi as fingerprint is another widely deployed indoor localization scheme [2]. It consists of two phases, site survey (or training phase) and location lookup. In site survey, the signal strengths from different Wi-Fi access points (APs) are measured and recorded. This creates a 2-D "heat" or "fingerprint" map of signal strength for each AP. In the lookup phase, an MS measures the signal strengths of the APs. They are compared against those on the map, and the location is computed accordingly. Fingerprinting is fast. Its main source of error is that at the same location, site-surveyed RSS may be different from that obtained by MS in the lookup phase.

It is clear from above that indoor localization techniques have their own errors. In order to achieve better localization accuracy, we need to first understand how their errors depend on system parameters and measurement noises. With this understanding, we are able to configure and optimize the system to achieve better accuracy.

Traditionally, indoor localization techniques are studied in "isolation," i.e., algorithms are studied and applied specifically for one localization scheme. In reality, many of these localization techniques may exist at the same time, so an MS may receive many estimations on its location from different deployed techniques (such as Wi-Fi, RFID, INS, Ultrasonic, etc). We hence need to consider how to properly combine these estimations, the so-called *estimation fusion*, to overcome their own estimation errors and achieve better localization.

Estimation fusion is a technique on how to best utilize information embedded in multiple sets of data (usually obtained from multiple sources) to estimate an unknown quantity [3]. When it is applied to localization, the output of multiple estimators with different location errors are combined, or "fused," to produce an estimation which is more accurate than any of the constituent estimators. The problem is designing the optimal algorithm to fuse these

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estimations to achieve maximum accuracy.

Our contributions of this work are as follows:

- *Modeling the estimation error of localization techniques:* We present analytic models on the localization error of two major localization techniques using RFID and Wi-Fi (Fingerprinting). Our models are realistic, capturing important system parameters and measurement noises, and quantifying how they affect the accuracies of the location of MS.
- *Estimation fusion and its optimal analytic closed-form solutions:* Given estimation errors of co-existing localization techniques at an MS, we formulate the problem of optimal fusion to achieve higher estimation accuracy. We present closed-form and computationally-efficient solutions. Our simulated and experimental results show that our fusion approach can optimally combine the output of different estimators to achieve much lower estimation errors. It effectively attains high accuracies even though the constituent estimators are noisy.

The paper is organized as follows. After reviewing previous work on indoor localization in Section II, we present models on estimation errors for RFID and Wi-Fi in Sections III and IV, respectively. Given estimation errors, we present our fusion algorithm in Section V. Numerical and experimental results for fusion of RFID and Wi-Fi are shown in Section VI. The conclusion is made in Section VII.

## II. Related Work

We briefly review related work here. Localization errors for techniques such as AOA, TOA, TDOA and RSS have been analyzed and discussed in [4], [5]. We present here models for RFID and fingerprinting, which have not been analyzed before.

Many localization techniques have been proposed and studied (see, for examples, [1], [6], [7]). While these works are impressive, the proposed schemes are studied in "isolation." We, on the other hand, study how to *fuse* different techniques by optimally combining them to achieve better accuracy.

Estimation fusion has traditionally been applied in signal processing [3]. In the context of localization, the work in [8] proposes a cooperative positioning system that combines TDOA and RSS by means of non-linear least-squares fusion algorithm. The work in [9] discusses "XINS" which combines INS and any other technique "X." All these works are to fuse two estimations and the fusion algorithms are mostly heuristic in nature. We consider the general case of fusing any arbitrary number of estimations, and derive a computationally-efficient closed-form solution to optimally combine them.

## III. Modeling Location Error for RFID

In this section, we analyze location error of LANDMARC, which is an indoor localization scheme using RFID technique. As we have mentioned, there are readers and active tags (as reference/target) in this system. The positions of readers and reference tags are fixed. When MS (with tag attached) stands at some position, readers obtain received signal strength from reference tags and the MS simultaneously such that for MS and each reference tag, a signal vector of readings from different readers can be constructed. By comparing the similarity between signal vectors from the MS and reference tags, position of the MS can be estimated using a weighted sum of positions from reference tags.

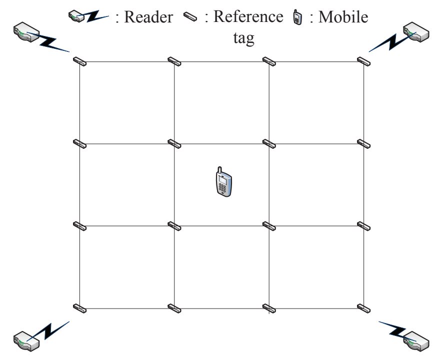


Figure 1: Topology of LANDMARC localization scheme.

Suppose we set up $R$ readers labelled from $1$ to $R$, $T$ reference tags, with positions at coordinates $(x_1, y_1), ..., (x_T, y_T)$, and one MS (attached with an active tag), with position coordinates $\mathbf{X} = (x, y)$ in an area. Denote received signal strength of the $t$th reference tag at the $r$th readers as $\mathbf{\Theta}_{tr}$ and that of mobile as $\mathbf{S}_r$. The system works as follows. First, measurements of signal strength $\mathbf{\Theta}_{tr}$'s and $\mathbf{S}_r$'s are obtained. Let $D_k$ be the Euclidean distance between signal strengths from reference tag $k$ and MS, which is given by $D_k^2 = \sum_{r=1}^{R}(\Theta_{kr} - S_r)^2$. The position of MS is then calculated as a weighted sum of positions of $K$ nearest reference tags (neighbors) in terms of signal vector distance, i.e.,

$$(\hat{x}, \hat{y}) = w^T X = \sum_{k=1}^{K} w_k (x_k, y_k), \tag{1}$$

where $w_k$ is the weight given by

$$w_k = \frac{1/D_k^2}{\sum_{i=1}^{K} 1/D_i^2}.$$

We consider that $D_k$ has measurement noise. Given that signal strength is usually expressed in dBm, we write $D_k^{dBm} = \mu_k + N(0, \sigma^2/4)$, where $\mu_k$ is the mean of the measurement in dBm and $\sigma/2$ is the measurement noise in dBm. This simply means that $D_k$ follows log-normal distribution. Let $z_k = 1/D_k^2$, which is clearly log-normal with mean $-2\mu_k$ and variance $\sigma^2$. Given that, then $w_k$ can be written as

$$w_k = \frac{z_k}{\sum_{i=1}^{K} z_i}. \tag{2}$$

For summation of log-normal distribution in the denomi-

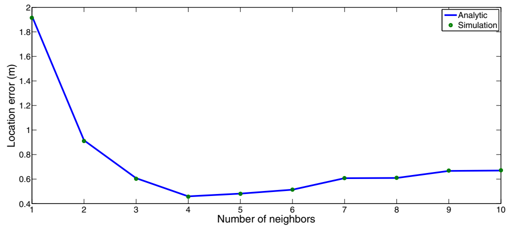


Figure 2: Location error versus number of neighbor in LANDMARC.

nator of Equation (2), there is no closed-form distribution to model it. Normally the sum is approximated by another log-normal distribution. Here, we use Fenton-Wilkinson method [10] to approximate it. This method is accurate for dB spread less than 4, which corresponds to $\sigma$ here in around $[0, 1]$. We then have

$$Z = \sum_{k=1}^{K} z_k \sim \text{log-}N\left(\mu_Z, \sigma_Z\right), \tag{3}$$

$$\sigma_Z^2 = \log\left[\frac{\sum_{k=1}^{K} e^{2\mu_k+\sigma^2}\left(e^{\sigma^2}-1\right)}{\left(\sum_{k=1}^{K} e^{\mu_k+\sigma^2/2}\right)^2}+1\right], \tag{4}$$

$$\text{and} \quad \mu_Z = \log\left[\sum_{k=1}^{K} e^{\mu_k+\sigma^2/2}\right] - \frac{\sigma_Z^2}{2}. \tag{5}$$

Thus $w_k = z_k/Z$ is also a log-normal distribution with

$$\begin{aligned} \mu_{w_k} &= -\mu_Z + \mu_k, & (6)\\ \sigma_{w_k}^2 &= \sigma_Z^2 + \sigma^2, & (7)\\ E(w_k) &= e^{\mu_{w_k}+\sigma_{w_k}^2}, & (8)\\ \text{and} \quad Var(w_k) &= \left(e^{\sigma_{w_k}^2}-1\right)e^{2\mu_{w_k}+\sigma_{w_k}^2}. & (9) \end{aligned}$$

Finally, the square of location error is

$$\begin{aligned} MSE &= Var(\hat{x}) + (E(\hat{x}-x))^2 & (10)\\ &= \sum_{k=1}^{K}\left[x_k^2 Var\left(w_k\right)\right] + \left[\left(\sum_{k=1}^{K} x_k E(w_k)\right) - x\right]^2. \end{aligned}$$

We have done simulations to study our model. The simulation is implemented in an area of $g \times g$ grids with reference tags $s$ meters apart, as shown in Figure 1. Readers are placed at the circumference centered at the grid center $(0, 0)$ with radius $r$. Location of Reader $i$ is $(r\cos(i2\pi/NR), r\sin(i2\pi/NR))$. Unless stated otherwise, we use the default parameters $(R, K, \sigma_i, g, s, r)$ = $(4, 4, 0.05, 5, 5, 12.5\sqrt{2}+5)$. Here we are interested in two important system parameters, $R$ and $K$.

We show in Figure 2 location error versus number of neighbors. Clearly, as number of neighbors increases, location error decreases. It is because new neighbors can "pull" the estimation towards the right direction. There is some fluctuation at the tail part. The reason is neighbors farther away may bring more error than correction to the estimation if weight is not correctly assigned. Thus, in real deployment, a few neighbors is enough. Having too many neighbors may not be good. From the plot, we can also see that the analytic curve fits simulation results closely.

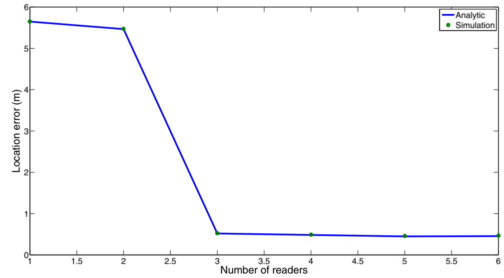


Figure 3: Location error versus number of readers in LANDMARC.

In Figure 3, we show location error versus number of readers. As number of readers increases, location error decreases. This is because weight calculation is more accurate with more readers placed. There is a strong threshold in the number of readers. It is due to great increment in the weight of the nearest neighbors. Where it happens depends on the method of placing readers. As we can see, the estimation has not improved much later, thus, in practice, we may want to find the number of readers whose marginal benefit is still high.

## IV. Modeling Location Error for Fingerprinting

In this section, we analyze location error of Fingerprinting. There are two phases in this system: site survey and location lookup. After finishing site survey, when the signal strengths of APs are obtained by MS, we look up locations from the "heat" map, one location for each AP. All these locations are then combined to locate the MS (e.g., using trilateration). We would like to minimize the least square error in trilateration formulation. Suppose there are $m$ Wi-Fi sources covering an indoor area, and a MS is located at $\mathbf{X} = (x, y)^T$ within the area. For the MS, it will obtain a signal profile $(r_1, ..., r_m)$ through measurement. Based on this profile, without loss of generality, we assume $m$ Wi-Fi sources will give $n$ location estimations $(\hat{\mathbf{X}}_1, ..., \hat{\mathbf{X}}_n)$ for the MS, with $\hat{\mathbf{X}}_i = (\hat{x}_i, \hat{y}_i)$. Thus the distance $\hat{d}_i$ between the $i$th Wi-Fi source and the estimated MS location $\hat{X}_i$ can be obtained. Denote the positions of the $i$th Wi-Fi source as $(a_i, b_i)$, we have $\hat{d}_i^2 = (\hat{x}_i - a_i)^2 + (\hat{y}_i - b_i)^2$, and $\Delta\hat{d}_i^2 = \hat{d}_1^2 - \hat{d}_{i+1}^2$. The system of equations can be written succinctly as

$$\mathbf{H}\hat{\mathbf{X}} = \hat{\mathbf{B}}, \tag{11}$$

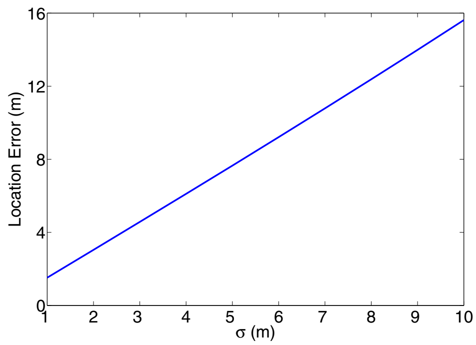


Figure 4: Location error versus distance measurement noise for Fingerprinting.

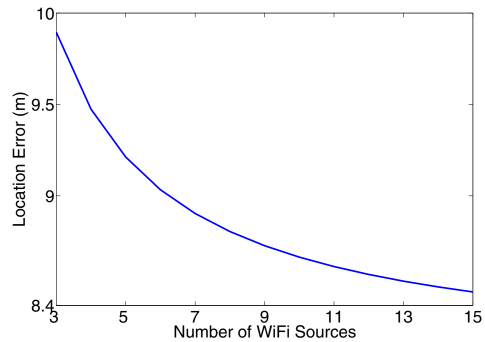


Figure 5: Location error versus number of Wi-Fi sources for Fingerprinting.

where $\mathbf{H}$ is an $(n-1)\times 2$ matrix and $\hat{\mathbf{B}}$ is an $(n-1)\times 1$ matrix, whose $i$th row are given by

$$\begin{aligned} \mathbf{H}_i &= [a_{i+1}-a_1, b_{i+1}-b_1], \\ \text{and} \quad \hat{\mathbf{B}}_i &= \left(\Delta\hat{d}_i^2 + (a_{i+1}^2+b_{i+1}^2) - (a_1^2+b_1^2)\right)/2. \end{aligned}$$

Solving for least-squared error, we then have

$$\hat{\mathbf{X}} = (\mathbf{H^T H})^{\dagger}\mathbf{H^T}\hat{\mathbf{B}}. \tag{12}$$

Thus,

$$Cov(\hat{\mathbf{X}}) = (\mathbf{H^T H})^{\dagger}\mathbf{H^T \Sigma H}(\mathbf{H^T H})^{\dagger}, \tag{13}$$

where

$$\begin{aligned} \mathbf{\Sigma}_{ij} &= E\left(\left(\Delta\hat{d}_i^2 - (E\hat{d}_i^2 - E\hat{d}_{i+1}^2)\right)\right. \\ &\times \left.\left(\Delta\hat{d}_j^2 - (E\hat{d}_1^2 - E\hat{d}_{j+1}^2)\right)\right). \end{aligned} \tag{14}$$

We study location error by considering normal noise on $d_i$, with $\hat{d}_i \sim N(d_i, \sigma^2)$. Wi-Fi sources are equally spaced on the circumference of a circle with radius $r$ (meters). Without otherwise stated, $(r, n, \sigma) = (30, 5, 6)$.

In Figure 4, we plot location error (given by Equation (13)) versus $\sigma$. The error increases quite fast as $\sigma$ increases, so it has a major influence on the system's performance. This can also show the intuition to design fingerprinting systems, since the map will greatly reduce $\sigma$ and improve accuracy in localization.

We plot in Figure 5 location error versus number of Wi-Fi sources. It is obvious that with the increase in $n$, the estimation error for $x_0$ will be reduced. However, as $n$ increases, the marginal benefit of Wi-Fi sources is diminishing because there is correlation in estimation among different sources. This is a factor that needs to be carefully considered in real deployment, since in designing a system, we would like to optimize the cost of resources.

## V. Optimal Estimation Fusion

We study in the following section how to combine, or fuse estimations from multiple localization schemes to achieve better accuracy, provided their variance is known. A closed form optimal solution is derived and the scheme's performance is shown. The major difference between the work here and Kalman Filter is that the process we study is not necessarily stationary, thus we do not need the assumption that the measurement noise follows Gaussian or any other distribution.

Though we consider 1D case here, the arguments can be easily extended to higher dimension. Suppose the MS is located at $x_0$. Denote estimations from $n$ different schemes on MS position as $(\hat{X}_1, \ldots, \hat{X}_n)$, where $\hat{X}_i$'s are independent random variables due to uncertainties in location estimation. We model $\hat{X}_i$ to follow some distribution with mean $x_0$ and standard deviation $\sigma_i$ for $i \in \{1, \ldots, n\}$. It is reasonable to assume the fused estimator $\hat{X}$ is in the convex hull formed by $(\hat{X}_1, \ldots, \hat{X}_n)$, i.e.,

$$\hat{X} = \sum_{i=1}^{n} \beta_i \hat{X}_i, \tag{15}$$

where $\sum_{i=1}^{n}\beta_i = 1$. Thus, $\hat{X}$ is also an unbiased estimator, which means $E(\hat{X}) = x_0$. Then, we would like to make the estimation more accurate by minimizing its output distance from $x_0$ every time, i.e., the variance of $\hat{X}$. Note that our analysis is not based on any assumption of the distribution type. Let $\beta = (\beta_1, \ldots, \beta_n)^T$. By previous analysis, we formulate our problem as minimizing the variance of $\hat{X}$ with

$$Var(\hat{X}) = Var\left(\sum_{i=1}^{n}\beta_i\hat{X}_i\right) = \beta^T \text{diag}(\sigma_i^2)\beta, \tag{16}$$

where $\text{diag}(\sigma_i^2)$ is an $n \times n$ diagonal matrix with diagonal entries being $\sigma_i^2$. Thus, we seek to find the optimal $\beta$ to solve the following minimization problem:

$$\begin{aligned} \underset{\beta}{\text{minimize}} \quad & \beta^T \text{diag}(\sigma_i^2)\beta \\ \text{subject to} \quad & ||\beta||_1 = 1, \\ & \beta \geq 0, \end{aligned} \tag{17}$$

where $||\beta||_1$ is the $l_1-$ norm of $\beta$. This problem is convex and the closed-form optimal solution is

$$\beta_i^{\star} = \left(\sigma_i^2 \sum_{j=1}^{n}\frac{1}{\sigma_j^2}\right)^{-1}, \tag{18}$$

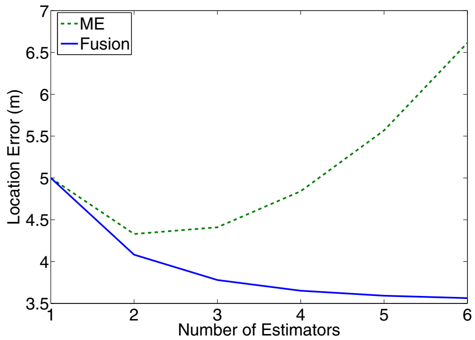


Figure 6: Location error versus number of estimators.

which can be obtained by using Lagrange multiplier. This optimal solution is quite intuitive since for accurate estimators, we would always like to weigh it more. Thus, the optimal variance is

$$Var^{\star}(\hat{X}) = \sum_{i=1}^{n} \left( \sigma_i \sum_{j=1}^{n} \frac{1}{\sigma_j^2} \right)^{-2}. \tag{19}$$

## VI. Numerical and Experimental Results

In this section, we present illustrative numerical and experimental results for our fusion algorithm.

### A. Numerical Results

We present below illustrative simulation results on evaluating our fusion algorithm. In our simulation, there are $n$ estimators labelled $i = 1, 2, \ldots, n$. Without loss of generality, we consider their estimation variance in increasing order as

$$\sigma_i^2 = \alpha^{i-1} \sigma_0^2, \tag{20}$$

for some $\sigma_0$ (meters) and $\alpha \geq 1$.

We compare our algorithm with the traditional mean estimator (ME), where the mean of estimations from different estimators is used as its estimation, i.e., $\hat{X}_{ME} = \sum_{i=1}^{n} \hat{X}_i / n$. Therefore, its variance $Var(\hat{X}_{ME})$ is

$$\begin{aligned} Var(\hat{X}_{ME}) &= \frac{1}{n^2} \sum_{i=1}^{n} Var(\hat{X}_i) \\ &= \frac{1}{n^2} \sum_{i=1}^{n} \sigma_i^2. \end{aligned} \tag{21}$$

Clearly, individual estimations $\hat{X}_i$ for all $i$ plays an equally important role, and estimation $\hat{X}_i$ with large $\sigma_i$ greatly increases $Var(\hat{X}_{ME})$.

There are two parameters of interest, namely $(n, \alpha)$. Without otherwise stated, $(n, \sigma_0, \alpha) = (5, 5, 2)$. Variance of our fusion scheme, single estimator and ME are generated based on Equation (19), (20) and (21), respectively.

The location error with respect to $n$ is plotted in Figure 6. With more estimators employed, location error of ME gets larger since individual estimations get more inaccurate according to Equation (20). However, the location error of our fusion algorithm drops in the same situation. This indicates that, as long as the estimator has correct mean value, even if its variance is extremely large, with our algorithm, it can still help to improve the localization accuracy by shifting more weights to more accurate estimators, which is quite promising.

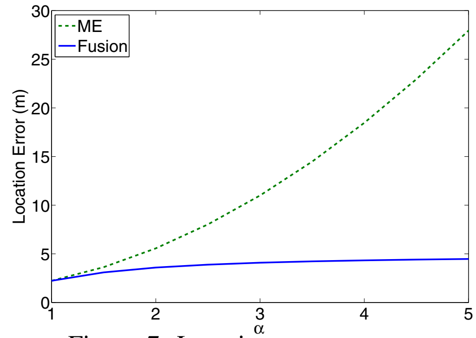


Figure 7: Location error versus $\alpha$.

Figure 7 shows the location error with respect to $\alpha$. As $\alpha$ increases, the estimation error of ME grows faster and faster, and clearly unbounded above. Our fusion algorithm, although still growing, converges to the smallest $\sigma$. It shows that the fusion is effective, provided the error of location is estimated correctly.

### B. Experimental Results

We have conducted a set of experiments in our lab to compare the results from Fingerprinting (FP), RFID and fusion. Algorithms used are according to previous sections. The room map, together with placement of RFID readers, reference tags and targets, is shown in Figure 8. We use RFID devices from RF code, including two identical M220 readers and 14 identical active tags (ten as references and four as targets) operating at 433M Hz. We also have four access points installed for FP. Four Android Nexus phones are placed at the same positions as target tags to collect Wi-Fi signals for FP. Table I shows the location information.

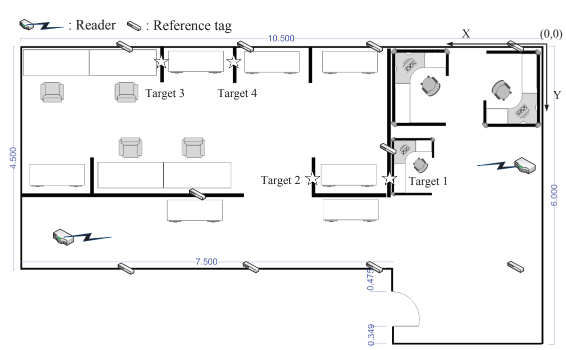


Figure 8: Experiment setup.

Table I: Location information of devices.

| AP/Reader/Reference Location | | | |
|---|---|---|---|
| AP 1 | (10.25, 0.25) | Reference 3 | (5.75, 4.5) |
| AP 2 | (0.25, 2.75) | Reference 4 | (8.25, 4.5) |
| AP 3 | (6.25, 11.25) | Reference 5 | (3, 2.25) |
| AP 4 | (9.25, 12.25) | Reference 6 | (6.75, 3) |
| Reader 1 | (8.75, 3.75) | Reference 7 | (0.75, 0) |
| Reader 2 | (0.25, 3.25) | Reference 8 | (3.25, 0) |
| Reference 1 | (0.75, 4.5) | Reference 9 | (5.75, 0) |
| Reference 2 | (3.25, 4.5) | Reference 10 | (8.25, 0) |

In the experiment, we collect RSS data from APs and RFID readers at the same time for ten seconds. Estimations of targets' positions are made every two seconds. For RFID, number of neighbors used is three in Equation (2). After obtaining all five estimations from a single scheme, we calculate the variance of them and use the variance to calculate the weight for each scheme by Equation (19), then get an estimation for fusion by $\hat{x}_{\text{fusion}} = \text{weight}_{\text{FP}} \times \hat{x}_{\text{FP}} + \text{weight}_{\text{RFID}} \times \hat{x}_{\text{RFID}}$.

The experiment is repeated numerous times until the statistics are stable. We define $MSE = \sum_{i=1}^{n} \left( (\hat{x}_i - x)^2 + (\hat{y}_i - y)^2 \right) / n$, where $(x, y)$ is the true location. We use $\sqrt{MSE}$ as a metric for accuracy. Figure 9 shows the experiment results. We can tell from the results that fusion tends to place more weight on more accurate scheme and achieves better estimation. For Target 3 and Target 4, the error is quite high, this is because the cubical environment in the lab blocks line of sight transmission for both RFID and Wi-Fi signals.

## VII. Conclusion

Indoor multimedia LBS is drawing much attention in industry due to its strong growth and commercial potential. Its success largely depends on accurate indoor localization. In this paper we first analyze the estimation error of two important indoor localization techniques using RFID and Wi-Fi. Our models are realistic, capturing various important system parameters and measurement noises, and quantifying how they affect the accuracies of the techniques. We have also proposed an estimation fusion scheme to better estimate mobile device position in indoor environment. We have derived a closed-form solution to optimally combine estimations from different techniques and analyzed its sensitivity on the accuracy of individual estimators. Both numerical and experimental results show that our algorithm achieves lower estimation error than traditional mean or individual estimators.

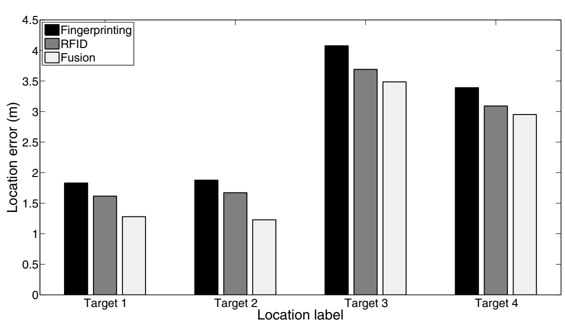


Figure 9: Experiment results.


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