Architecture-Intact Oracle for Fastest Path and Time Queries on Dynamic Spatial Networks

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ABSTRACT

Given two vertices of interest (POIs) s and t on a spatial network, a distance (path) query returns the shortest network distance (shortest path) from s to t. This query has a variety of applications in practice and is a fundamental operation for many database and data mining algorithms.

In this paper, we propose an efficient distance and path oracle on dynamic road networks using the randomization technique. Our oracle has a good performance in practice and remarkably, and at the same time, it has a favorable theoretical bound. Specifically, it has $O(n \log^2 n)$ (resp. $O(n \log^2 n)$) preprocessing time (resp. space) and $O(\log^4 n \log\log n)$ (resp. $O(\log^4 n \log\log n + l)$) distance query time (resp. shortest path query time) as well as $O(\log^3 n)$ update time with high probability (w.h.p.), where n is the number of vertices in the spatial network and l is the number of edges on the shortest path. Our experiments show that the existing oracles suffer from a huge updating time that renders them impractical and our oracle enjoys a negligible updating time and meanwhile has comparable query time and indexing cost with the best existing oracle.

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1 INTRODUCTION

With the advance of GPS technology and the prevalence of mobile devices, obtaining *real-time* spatial data becomes increasingly popular, and many existing commercial companies such as "Google Maps" and "Bing Maps" are currently using real-time spatial data for finding the *fastest* path from a given source to a given destination. The fastest path is computed based on a *dynamic* spatial network, where the weight associated to each edge could denote the travel time on this edge and usually changes over time (due to the traffic on this edge). Figure 1 shows a snapshot of a dynamic spatial network, which is a directed graph with 8 vertices, namely $v_1, v_2, ..., v_8$, each denoting a spatial location and 10 edges each denoting a road segment. In this snapshot, 3 edges have their weights = 2 and the other edges have their weights = 1. Let n be the total number of vertices in the spatial network.

The fastest path and the shortest travel time from a given source to a given destination on a dynamic spatial network could be regarded as the shortest path and the shortest distance when the weight of each edge is regarded as "distance" instead of "travel time". In the following, we refer them as "shortest path" and "shortest distance".

Computing the shortest path and the shortest distance from a source to a destination on a dynamic spatial network is very important. The reason is that there are a lot of applications starting from traditional drivers' navigation, which "aids" drivers to find a path, to recent autonomous car navigation, which "completely control" unmanned vehicles to move on the road. Besides, computing them *efficiently* is very important in some real-life applications. One example is emergency applications like delivering ambulances and fire engines to target areas. In Hong Kong, it is expected that ambulances arrive to target areas within about 12 minutes [5]. Another example is time-critical applications in which business people and travel salesmen with their own tight schedule would prefer their travel schedule as fast as possible without any delay. Furthermore, "shortest path/distance" is a fundamental operator of many spatial queries, such as nearest neighbor queries, range queries and spatial join queries, and is also used in many scientific simulations [12, 33, 47].

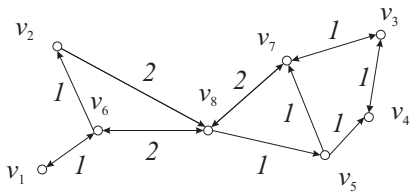


Figure 1: An Example of A Spatial Network

Returning the shortest path and the shortest distance efficiently based on a dynamic spatial network is very challenging. Firstly, re-computing the shortest path and the shortest distance from scratch is very costly whenever the weight of an edge in the spatial network is updated. A straightforward implementation is to execute the Dijkstra's algorithm [27] based on the current snapshot of the spatial network and obtain the shortest path/distance whenever there is a weight update. Although it is simple to implement, it is still slow and does not return a result efficiently.

Secondly, existing algorithms [7, 8, 13, 29, 31, 38, 50–55, 63] originally designed for finding the shortest path/distance on a *static* spatial network, only one snapshot of a spatial network, pre-compute some results in the form of index (called *oracle*) and return the shortest path/distance efficiently. But, they do not support the update operation on the pre-computed results due to weight update. Thus, they could not be used efficiently in the dynamic spatial network setting. A straightforward adaptation of these algorithms for the dynamic network setting is to re-construct the pre-computed results from scratch whenever there is a weight update. Then, each shortest path/distance query could be answered based on the re-constructed pre-computed results. Since the re-construction time complexities of these algorithms are worse than $O(n^{1.5})$, they are not efficient enough for handling shortest path/distance queries [61, 63].

Thirdly, there are existing algorithms [26, 30, 56] proposed for answering shortest path/distance queries on a dynamic spatial network each of which also constructs an index/oracle for answering shortest path/distance queries efficiently. But, for the sake of maintaining the index/oracle due to the weight update on the dynamic spatial network, the empirical update time of each of these algorithms is still large. Even if some of the algorithms have theoretical upper/lower bounds on update time complexities, the lower bounds are also at least linear to the number of vertices in the network. In practice, the number of vertices is very large. Thus, those algorithms are not scalable to the large spatial network.

Motivated by this, in this paper, we propose an oracle called **Update Efficient (UE)** that has a short update time and has a poly-logarithmic update time complexity, the first best-known result. Besides, this oracle returns the shortest path/distance efficiently. The key idea of the update efficiency of *UE* is to keep the *architecture* (or *structure*) of *UE* *unchanged* (or *intact*) for any weight update on the dynamic spatial network. All existing oracles require to perform a time-consuming operation of updating/adjusting the architectures of their oracles for any weight update. The reason is that the architecture of *UE* is dependent on only the original graph/network structure (without the weight information) and the *initial* randomly assigned ordering on vertices on the graph/network. These remain *unchanged* for any weight update. In contrast, the architecture of existing oracles in the literature is dependent on the *changing* weight of any edge on the spatial network. We will elaborate more in Section 4.

We summarize our major contributions as follows. Firstly, we propose a novel oracle called **UE** that answers shortest path/distance queries efficiently. Secondly, the time complexity of updating our oracle is $O(\log^3 n)$. This is the first best-known theoretical result since the existing best-known *worst-case* update time complexity is at least $O(n^{1.5})$ and the existing best-known *lower bound* on the update time complexity of the existing oracles is at least $\Omega(n)$. Thirdly, our empirical study shows that the update time of all existing oracles are too large, which could not be used in practice, but the update time of our oracle is very small. In particular, most existing oracles need to take more than one week for weight update, and the update time of our oracle (which is at most 50ms) is several orders of magnitude shorter than that of the best-known oracle on all networks tested.

The remainder of the paper is organized as follows. Section 2 defines the problem and Section 3 gives the related work. Section 4 presents our distance oracle. Section 5 shows our experimental results and Section 6 concludes our work.

2 PROBLEM DEFINITION

Consider a spatial network $G(V, E)$, where V is a set of all vertices and E is a set of all edges on the network. Let $n = |V|$ and $m = |E|$. Each directed edge $(u, v) \in E$ in G has its origin u and its destination v. Besides, it is associated with its *weight*, denoted by $w_G(u, v)$, which changes over time. Given a network $G(V, E)$ and a vertex $u \in V$, we define the set of *in-neighbors* of u, denoted by $N_G^{in}(u)$, to be the set of all vertices that are involved in edges in E with destinations as u. That is, $N_G^{in}(u) = \{v \in V | (v, u) \in E\}$. Similarly, we define the set of *out-neighbors* of u, denoted by $N_G^{out}(u)$ to be the set of all vertices that are involved in edges in E with origins as u. That is, $N_G^{out}(u) = \{v \in V | (u, v) \in E\}$. In this paper, we assume that there is no isolated vertex (i.e., for each $u \in V$, $N_G^{in}(u) \cup N_G^{out}(u) \neq \emptyset$).

Given two vertices, namely s and t, in V, we define a *path* from s to t in the form of a list of vertices, namely $(v_1, v_2, ..., v_l)$, such that $v_1 = s$, $v_l = t$ and for each $i \in [1, l-1]$, $(v_i, v_{i+1}) \in E$. Given a path π in the form of $(v_1, v_2, ..., v_l)$, we define the *weight* or the *length* of π to be $\sum_{i=1}^{l-1} w_G(v_i, v_{i+1})$.

In the above, we represent the path in the form of a list of vertices, says $(v_1, v_2, ..., v_l)$, where $(v_i, v_{i+1}) \in E$. We could also represent the path as a list of consecutive edges in the form of $(v_1, v_2)\cdot(v_2, v_3)\cdot\ldots\cdot(v_{l-1}, v_l)$. We call all vertices along this path that are not a source vertex and are not a destination vertex as *passing vertices*. In this example, $v_2, v_3, ..., v_{l-1}$ are passing vertices but v_1 and v_l are not.

Given a path π from a vertex s to another vertex t in V, π is said to be the *shortest path* from s to t in network G, denoted by $s \rightarrowtail_G t$, if the length of π is the smallest among all possible paths from s to t in G. Similarly, the *shortest distance* from s to t in network G, denoted by $d_G(s, t)$, is defined to be the length of $s \rightarrowtail_G t$. Note that the triangle inequality holds under function $d_G(\cdot, \cdot)$.

Problem 1 (Oracle). *We would like to design an oracle O that supports a short update time, and at the same time, answers any shortest path/distance query efficiently.*

In the case when there are multiple shortest paths from s to t, the shortest path query studied in this paper only finds an arbitrary shortest path among them. Thus, in the rest of this paper, the term 'shortest path from s to t' or 's-t shortest path' refers to an arbitrary shortest path from s to t.

In this paper, we just focus on studying the case when the weights of edges change over time. We do not consider the case when new vertices and new edges are added. The reason is that re-constructing the whole oracle from scratch is already enough. Specifically, this case means that new junctions and new road segments have to be built, which may take more than several weeks and even several years [6]. Since in our experimental results, each oracle could be built within one day, it is sufficient to rebuild the oracle from scratch in this case. Furthermore, we do not discuss the case when vertices and edges are removed. The reason is that this case refers to our case that the weights of the corresponding edges are updated to a very large value [28, 35, 49].

3 RELATED WORK

In this section, we first present the related work on *static* spatial networks (Section 3.1) and *dynamic* spatial networks (Section 3.2). Then, we describe some other studies that are related to our study in Section 3.3.

3.1 Oracle on Static Spatial Networks

There are a lot of existing studies about pre-computing oracles on static spatial networks. We categorize them into the following 7 categories: (1) *partition-based approaches* [37, 38], (2) *landmark-based approaches* [31], (3) *spatial coherence-based approaches* [50, 55], (4) *transit vertex-based approaches* [8, 13, 14, 61], (5) *2-hop labeling approaches* [7, 10, 21, 36], (6) *hierarchy-based approaches* [29, 51, 52, 63], and (7) a *hybrid approach* [46], namely *Hierarchical 2-Hop Index (H2H in short)*.

The partition-based approaches and landmark-based approaches are heuristic-based approaches without theoretical guarantee. For spatial coherence-based approaches, there are two representative algorithms under this category with theoretical bounds, namely *Spatially Induced Linkage Cognizance (SILC)* [50] and *Path-Coherent Pair Decomposition (PCPD)* [55]. For transit vertex-based approaches, one representative algorithm with theoretical bounds is called *Transit Vertex Variant (TVV)* [8]. For 2-hop labeling approaches, one representative algorithm is called *Pruned Landmark Labeling* (PLL) [10]. For hierarchy-based approaches, there are three representative approaches, namely the *Contraction Hierarchy (CH)* [29], the *Highway Hierarchy (HH)* [51, 52] and the *Arterial Hierarchy (AH)* [63]. The experiments of [9] show that CH performs better than PLL in terms of pre-computation time and space. For the Hierarchical 2-Hop Index (H2H in short), this algorithm combines the *2-hop labeling approach* and the *hierarchy-based approach*. This combination significantly reduces the time of finding labels in the 2-hop approach (used for finding the shortest distance) with the help of the hierarchy-based approach for a shortest distance query.

Although all of the above approaches were originally designed for shortest path/distance queries in the static spatial network setting (not the dynamic setting), we would like to adapt some of the approaches/oracles in the dynamic setting. Following [63], we consider adapting only oracles with theoretical analysis. The reason is that in the experimental results of [11, 34, 46, 61, 63], oracles with theoretical analysis out-perform all oracles without theoretical analysis (including the partition-based approaches and the landmark-based approaches). Notice that in this section, we compare adapted existing oracles with our oracle in theory and thus, only consider the oracles with theoretical bounds. But, in the experimental section, we compared many other algorithms without theoretical bounds. In this paper, we adapt five oracles, namely SILC, PCPD, TVV, AH and H2H, to handle the dynamic setting, and call them as SILC-Adapt, PCPD-Adapt, TVV-Adapt, AH-Adapt and H2H-Adapt. It is not difficult to adapt these oracles, and details of these adaptions could be found in [59]. Table 1 shows a summary of these adapted oracles in terms of preprocessing time, space, shortest distance query time, shortest path query time and update time. In this table, the first five rows correspond to these 5 adapted oracles. Notice that the update time complexity of each of these oracles is at least $O(n^{1.5})$. Besides, [59] shows that the lower bound of the update time complexity of each of these oracles is $\Omega(n)$, which means that each of these oracles is not scalable to a large network since n could be very large.

Note that although PLL and CH (designed for the static version) described above have theoretical bounds, we did not

Table 1: Comparison of Distance and Path Oracles on Dynamic Spatial Networks

Oracle	Preprocessing Time	Space	Distance Query Time	Shortest Path Query Time	Update Time
SILC-Adapt [50]	$O(n^2 \log n)$	$O(n\sqrt{n})$	$O(l \log n)$	$O(l \log n)$	$O(n^2 \log n)$
PCPD-Adapt [55]	$O(n^2 \log n)$	$O(s^3 \cdot n)$	$O(l \log n)$	$O(l \log n)$	$O(n^2 \log n)$
TVV-Adapt [8]	$O(n^2 \log n \log \alpha)$	$O(\Lambda n \log n \log \alpha)$	$O(\Lambda^2 \log^2 n \log^2 \alpha)$	$O(\Lambda^2 \log^2 n \log^2 \alpha + l)$	$O(n^2 \log n \log \alpha)$
AH-Adapt [63]	$O(hn^2\lambda^2)$	$O(hn\lambda^2)$	$O(h\lambda^2 \log h + h\lambda^4)$	$O(h\lambda^2 \log h + h\lambda^4 + l)$	$O(hn^2\lambda^2)$
H2H-Adapt [46]	$O(n^{1.5})$	$O(n)$	$O(\sqrt{n})$	$O(l \cdot \sqrt{n})$	$O(n^{1.5})$
Dynamic-CH [26, 30]	$O(n^2 \log n)$	$O(n^2)$	$O(\sqrt{n} \log n)$	$O(\sqrt{n} \log n + l)$	$O(n^2 \log n)$
Dynamic-PLL [11]	$O(n^2 \log^2 n)$	$O(n\sqrt{n} \log n)$	$O(\sqrt{n} \log n)$	$O(l\sqrt{n} \log n)$	$O(n^2 \log^3 n)$
this paper	$O(n \log^2 n)$ w.h.p.	$O(n \log^2 n)$ w.h.p.	$O(\log^4 n \log \log n)$ w.h.p.	$O(\log^4 n \log \log n + l)$ w.h.p.	$O(\log^3 n)$ w.h.p.

Remark: l is the number of edges in the shortest path. h (resp. λ) is the height of the hierarchy that is equal to $\log \frac{\max_{u,v \in V} d_G(u,v)}{\min_{u,v \in V} d_G(u,v)}$ (resp. the *Arterial Dimension* of the spatial networks) and they are proposed in [63]. h (resp. λ) is at most 26 (resp. 100) in the datasets tested in [63]. α is defined to be $\frac{\max_{u,v \in V} d_G(u,v)}{\min_{u,v \in V} d_G(u,v)}$. λ and Λ are all $\Omega(\sqrt{n})$ in a grid spatial network like that in Manhattan District. Note that the complexities of some of the above oracles rely on a feature, namely *tree width*, of the spatial network. It is shown in [2] that the tree width of a spatial network is is as large as $\sqrt{n}$ in a square grid spatial network that is common in Manhattan district. Thus, we simply treat tree width as $O(\sqrt{n})$ in this paper. s is around 12 in practice.

adapt them because these two oracles were already adapted by some existing papers and will be described in the next section. Note that those of the 2-hop labeling approaches other than PLL, though with theoretical bounds, are not shown since they are significantly outperformed by PLL as shown in the experiments of [10].

3.2 Oracles on Dynamic Spatial Networks

Recently, there are studies about pre-computing oracles on dynamic spatial networks. There are four representative oracles. The first oracle is the dynamic version of oracle HH called *Dynamic-HH* [56]. But, there is no theoretical bound on Dynamic-HH. The second oracle is the dynamic version of oracle CH called *Dynamic-CH* [26, 30]. The experimental results of [26, 30] show that Dynamic-CH outperforms Dynamic-HH in terms of preprocessing time, space consumption, query time and update time. The third oracle is the dynamic version of oracle PLL called *Dynamic-PLL* [11]. The fourth oracle is a *bit parallel shortest path tree indexing structure (*BPL*)* [34] that is an indexing structure of answering shortest path/distance queries on a dynamic network. It dynamically maintains several *shortest path trees* rooted at a number of *high-degree* nodes. The query algorithm is a bi-Dijkstra's algorithm enhanced with some pruning operations based on the information of the shortest path trees. It has good empirical performance but has no theoretical analysis.

Note that among four oracles described above, only Dynamic-CH and Dynamic-PLL have theoretical bounds. Table 1 shows a summary about the complexities of these two oracles, Dynamic-CH and Dynamic-PLL, where the third-to-last and second-to-last rows correspond to these two oracles.

3.3 Some Other Related Studies

There are some other studies related to our problem although they are not exactly our problem. The first branch of related studies is shortest path/distance queries on a *time-dependent* spatial network [15, 24, 39, 45, 57, 58, 62]. In these studies, edge weights follow a *pre-defined* function that takes time as input and returns a value (e.g., traffic) as output. This is unrealistic in real world with "unexpected" events. The second branch of related studies is *approximate* oracles on a dynamic spatial network [53–55]. An approximate oracle returns an *approximate* distance/path for each shortest distance/path query but our oracle returns an *exact* distance/path. The third branch of related studies is *Distance Sensitivity Oracle* [18, 19, 44, 48, 60] that answers the shortest path/distance queries with vertex/edge failures on static networks (not dynamic networks studied in this paper).

4 DISTANCE AND PATH ORACLE

In this section, we describe our distance and path oracle called the *update-efficient (UE)* oracle. Specifically, in Section 4.1, we first give an important property called the *architecture-intact property*, which is the key to the efficient update time of *UE* and could not be satisfied by most (if not all) existing oracles. Then, we describe our *UE* oracle in Section 4.2. Next, we present the algorithms for preprocessing, shortest distance/path query processing and update processing in Section 4.3, Section 4.4 and Section 4.5, respectively. Section 4.6 presents the theoretical analysis of our oracle.

4.1 Concept: Architecture-Intact Property

Given a data structure, the *architecture* of this data structure is defined to be a set of all components in this data structure excluding all non-structural information (e.g., the weight information). Different data structures have different definitions of "architecture". For example, consider a data structure S that is a *weighted* graph containing 3 components, namely V, E and W. V is a set of vertices, E is a set of directed edges and W is a set of numbers each of which denotes the weight of an edge in E. Here, each vertex in V could be regarded as a *sub-component* of V and each edge in E could be regarded as a *sub-component* of E. The architecture of this data structure S is equal to the un-weighted graph containing V and E only. Consider another example. The data structure used in *Dynamic-CH* [26, 30] is a spatial network/graph inserted

with additional edges called *shortcuts*, where each edge is associated with a weight. Note that in *Dynamic-CH*, a shortcut from a vertex u to a vertex v is defined to be a new edge that denotes/corresponds to the shortest path from u to v and its weight is defined to the length of the corresponding path. The architecture of this data structure is the un-weighted version of the spatial network/graph inserted with additional edges, where there is no weight associated to each edge. Consider the third example. The data structure used in *BPL* [34] is a set of *(shortest path) trees*, where each edge in the tree is associated with a weight. The architecture of this data structure is the same set of trees but there is no weight associated to each edge in the trees.

With the concept of "architecture", we are ready to present the major results of our oracle *UE*. *UE* satisfies one interesting property called the *architecture-intact property*, which is the key of an efficient update of our oracle. Specifically, we say that an oracle constructed from a spatial network satisfies the *architecture-intact property* if the *architecture* of the data structure used in this oracle remains unchanged even when there is an update on the weight of an edge in the spatial network. Whenever there is a weight update, we just need to update the non-structural information (e.g., the weight information) in our oracle *UE* without updating the architecture of the data structure. Updating the weight information could be done efficiently since *UE* stores a mapping table that could help to efficiently find a set of sub-components to be updated for each edge if its associated weight is updated.

In our technical report [59], we show that most (if not all) adapted algorithms originally designed for static spatial networks (in Section 3.1) do not satisfy the architecture-intact property. The same is true for all algorithms originally designed for dynamic spatial networks (in Section 3.2), i.e., they do not satisfy the architecture-intact property. This explains why our oracle *UE* has a shorter update time compared with existing algorithms. For example, in *Dynamic-CH*, when there is a weight update, some new shortcuts have to be added to the architecture of the data structure used in *Dynamic-CH*. Besides, in *BPL*, when there is a weight update, some edges in the trees are removed and some other new edges are inserted into the trees. Besides, this observation is verified in our experimental results that most existing oracles need to take more than one week for weight update, and the update time of our *UE* oracle (which is at most 50ms) is several orders of magnitude shorter than that of the best-known oracle on real-life datasets. More importantly, none of the existing oracles can handle the update before the next batch of update arrives in our experimental results.

The reason why a data structure that does not satisfy the architecture-intact property has a larger update time is that for any weight update, it has to perform a costly *search* operation on which components in the data structure should have some new sub-components to be added and should have some existing sub-components to be removed.

Let us give some details of our oracle *UE* to give the major key idea why *UE* satisfies the architecture-intact property. The idea is based on the *novel* definition of "shortcut" used in *UE* that is independent of any update on the weight of an edge in the spatial network. That is, *UE* does not need to add or remove any shortcuts in the network even though there is an update on the weight of an edge in the network. To the best of our knowledge, we are the first to give a "nice" definition of "shortcut" such that there is no need to introduce any insertion/deletion of "shortcut" for any edge-weight update. Existing studies gave other definitions of "shortcut" that may introduce potential insertion/deletion of "shortcut" for any edge-weight update.

4.2 Our Oracle: UE

Specifically, *UE* involves three components, namely the *UE spatial graph*, the *weight mapping table* and the *occupant mapping table*. The UE spatial graph is an un-weighted graph that is exactly equal to the un-weighted version of the original graph G but there are additional edges in the UE spatial graph compared with the original graph G. The weight mapping table contains all the weight information about the edges in the UE spatial graph. Specifically, it is a set of entries each in the form of $((u, v), w)$, where (u, v) is an edge in the UE spatial graph G' and w is the weight of (u, v) in graph G' (i.e., $w_{G'}(u, v)$). Here, although G' is un-weighted, for the ease of description, when we write "the weight of an edge in G'", we mean that there is a value called the *weight* associated to this edge in the weight mapping table. The occupant mapping table contains all the information about a new concept called "occupant", which will be described later. The occupant mapping table is a set of entries each in the form of $((u, v), o)$, where (u, v) is an edge in the UE spatial graph G' and o is a vertex in V called the *occupant* of (u, v). Intuitively, *UE* uses additional edges called *shortcuts* with the help of the weight mapping table and the occupant mapping table to answer shortest distance/path queries efficiently.

Before we define "shortcuts", we first define the concept of "rank" and then the concept of "valley path". Initially, we perform Knuth shuffle [41] to obtain a random ordering/permutation I of vertices in V. With this ordering/permutation I, the *rank* value of each vertex v in V, denoted by $R_I(v)$, is defined to be its position of this ordering/permutation. Consider our running example containing 8 vertices as shown in Figure 1. Let the permutation I be $(v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8)$. The rank value of v_1 (i.e., $R_I(v_1)$) is equal to 1. The rank value of v_2 (i.e., $R_I(v_2)$) is equal to 2.

A path $(v_1, v_2, ..., v_l)$ is said to be a *valley path* if the rank value of each vertex along the path except the first vertex v_1

and the last vertex v_l is smaller than both the rank values of v_1 and v_l (i.e., each vertex along this path except s and t has its rank value smaller than $\min\{R_I(s), R_I(t)\}$). The reason why we call this path as a "valley path" is described as follows. In our terminology, we "interpret" that each vertex could be associated with a location on a mountain with a *height*. If the vertex has a larger rank value, it has a greater height. According to this interpretation, all vertices along the path except the first vertex and the last vertex have their smaller heights, which looks like a "valley". Notice that as a special case, when a path from s to t contains exactly two vertices (i.e., the path is (s, t)), this path is also a valley path. Consider our running example in Figure 1, where permutation I is $(v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8)$. Path (v_5, v_4, v_3, v_7) is a valley path since both the rank values of v_4 and v_3 are smaller than 5 ($= \min\{R_I(v_5), R_I(v_7)\}$). Path (v_5, v_7) is also a valley path since it contains exactly two vertices. Path (v_5, v_8, v_7) is not a valley path because the rank value of v_8 is greater than 5 ($= \min\{R_I(v_5), R_I(v_7)\}$).

Besides, we denote the *shortest valley path* from s to t under the permutation I by $s \overset{I}{\underset{G}{\rightsquigarrow}} t$. If the context of permutation is clear, we simply denote it by $u \rightsquigarrow_G v$. Consider our running example. Path (v_5, v_7) is the shortest valley path from v_5 to v_7. Thus, $v_5 \overset{I}{\underset{G}{\rightsquigarrow}} v_7 = (v_5, v_7)$. But, path (v_5, v_4, v_3, v_7) is not the shortest valley path from v_5 to v_7 since there exists a path from v_5 to v_7 (i.e., (v_5, v_7)), which has a smaller length. In the following, when we write "$u \rightsquigarrow_G v \setminus \{u, v\}$", we mean the corresponding path *excluding* vertices u and v.

Given two vertices, namely u and v, a *shortcut* from u to v is defined to be an edge from u to v in G' denoting the shortest valley path from u to v on the *original* spatial network G. The *weight* of a shortcut is defined to be the length of the corresponding path in G.

UE keeps a certain number of shortcuts in the UE spatial network such that it maintains the following property called the *shortcut property* from time to time (e.g., even after there is an edge weight update). This property is the major key to the efficient update time of *UE*.

Property 1 (Shortcut Property). *For each pair of vertices, namely u and v, such that there exists a valley path from u to v, G' must have a shortcut from u to v (i.e., edge (u, v)) with weight equal to the length of the shortest valley path from u to v on the original network G.*

With the concept of "shortcut", we are ready to elaborate why there is no need to insert or remove any shortcuts in the UE spatial network G' even though there is an edge weight update, and thus, *UE* satisfies the architecture-intact property. Roughly speaking, *UE* keeps a certain number of shortcuts in G'. Firstly, there is no need to remove any existing shortcut with an edge weight update. Since each shortcut (u, v) in G' corresponds to the shortest valley path in G, whenever there is an edge weight update, although the *exact* shortest valley path in G may change, the shortcut (u, v) is still kept in G' (though conceptually, it denotes a different shortest valley path in G after the weight update), and thus, there is no need to remove the shortcut. Secondly, there is no need to add any new shortcut with an edge weight update. The reason is that *UE* keeps the same set of shortcuts in G' even with an edge weight update. By using this set of shortcuts together with the original edges in the network, *UE* could still answer shortest distance/path queries efficiently since the spatial network has a small *expansion dimension* (whose definition will be given later). Intuitively, if the expansion dimension is small, the number of edges to be traversed on G' of *UE* is small, and thus, the query time is shorter.

It is good to know that maintaining the shortcut property could help the performance of *UE* as described above. One remaining issue is how our *UE* oracle could maintain the shortcut property *efficiently* when there is a weight update. In this property, we know that there is a component of "shortest valley path from u to v on the original network G", which requires computation. One straightforward implementation is to enumerate all valley paths and to find the shortest valley path. Note that this implementation is very costly since it is possible that the number of vertices involved along the shortest valley path could be *large*. Fortunately, our *UE* considers a special form of "shortest valley path" called "shortest ternary valley paths" each of which involves only 3 vertices only so that maintaining the shortcut property is efficient.

We say that a valley path from a vertex to another vertex is *ternary* if this path involves 3 vertices only. In other words, the ternary valley path from a vertex to another vertex is in the form of (v_1, v_2, v_3), where v_2 has the smallest rank values among all these 3 vertices. Note that there is only one passing vertex in a ternary valley path. In this case, v_2 is a passing vertex. Similarly, the shortest ternary valley path from a vertex u to another vertex v is defined to be the shortest path among all ternary valley paths from u to v.

Interestingly, we found that the length w of the shortest valley path from a vertex u to another vertex v is equal to the length of the shortest ternary valley path from u to v (if there is no edge (u, v) on G). If there is an edge (u, v) on G with its weight w', it is equal to $\min\{w, w'\}$. We formalize this property called the *ternary valley path property* as follows.

Property 2 (Ternary Valley Path Property). *Let π be the shortest ternary valley path from vertex u to vertex v on the UE spatial network G'. Let w be the length of this path π. Let w' be the weight of the edge (u, v) (if any) on the original graph G (if there is no edge (u, v) on G, w' is set to ∞). The shortest valley path from u to v on G has the length equal to* $\min\{w, w'\}$.

With this interesting property, maintaining the shortcut property becomes much efficient. Specifically, since the set of possible paths considered in *UE* (which are ternary valley paths) involving at most 3 vertices is much smaller than the set of possible paths (which are ternary/non-ternary valley paths) involving any number of vertices, the computation of *UE* is very efficient. The correctness of these 2 properties (i.e., the shortcut property and the ternary valley path property) will be shown in the later section.

Next, we present 3 phases of *UE*, namely the preprocessing phase (Section 4.3), the query phase (Section 4.4) and the update phase (Section 4.5).

4.3 Preprocessing Phase

The preprocessing phase of *UE* involves the initialization step and the construction step.

Consider the initialization step. *UE* has three components, namely the UE spatial network, the weight mapping table and the occupant mapping table. They have the following initialization steps. Initially, the UE spatial network G' is initialized to G. Besides, the weight mapping table is initialized to contain a list of $((u, v), w)$ for each (u, v) in E, where w is the weight of (u, v) in G. Furthermore, the occupant mapping table is initialized to be an empty list. In the following, when we update $w_{G'}(u, v)$ with value w', we mean that we update the entry (u, v) in the weight mapping table with the value w'. Similar arguments could be made to the occupant mapping table (instead of the weight mapping table). Notice that due to the later operation that we could update $w_{G'}(u, v)$ with another value w', the weight of edge (u, v) in G' (*initialized* to the weight of edge (u, v) in G) could be different from the weight of edge (u, v) in G.

The construction step involves the following two steps.

- **Step 1 (Vertex Rank Generation):** It assigns each vertex in the spatial network with a random unique rank value that is a positive integer in $[1, n]$ based on Knuth Shuffling [41].
- **Step 2 (Iterative Vertex Contraction):** It performs an important operation of "vertex contraction" of each vertex in the ascending order of their rank values on G'.

In the following, we first give a major idea of Step 2. Without loss of generality, let us assume that for each $i \in [1, n]$, v_i has its rank value equal to i (for illustrating the major idea of Step 2). Initially, G' is initialized to G. We perform an operation "vertex contraction" of v_i on G' in an order of its rank value from 1 to n, where $i \in [1, n]$. Here, we say that vertex v_i is *contracted* or a *contracted vertex* (if the vertex contraction operation of v_i has been performed). G' is being updated after each vertex contraction operation. Let $G'_{(0)}$ be G' before we perform any vertex contraction operation in Step 2. Let $G'_{(i)}$ be G' just after we perform the vertex contraction operation of vertex v_i for $i \in [1, n]$. In other words, $G'_{(i)}$ is a graph obtained after the vertex contraction operation of v_i on $G'_{(i-1)}$ for $i \in [1, n]$. For each $i \in [1, n]$, the vertex contraction operation of vertex v_i follows the following principle called the *Shortest Ternary Valley Path Maintenance Principle*. Note that the final G' obtained in Step 2 is equal to $G'_{(n)}$.

Principle 1 (Shortest Ternary Valley Path Maintenance Principle). *For each $i \in [1, n]$, for any two distinct vertices u and v in V, if there exists a* ternary *valley path from u to v on $G'_{(i-1)}$, where the (only one) passing vertex along this path is v_i, we perform the following operations.*

(1) ***Operation 1 (G' Update):*** *If edge (u, v) could not be found in $G'_{(i-1)}$, we create an additional edge (u, v) (called shortcut (u, v)) in $G'_{(i)}$. Otherwise, we do nothing (since we have edge (u, v)).*

(2) ***Operation 2 (Weight Update):*** *The weight of (u, v) on $G'_{(i)}$ (i.e., $w_{G'_{(i)}}(u, v)$) is set to $\min\{w, w'\}$, where w is the length of the shortest* ternary *valley path from u to v on $G'_{(i-1)}$ and w' is the weight of (u, v) on $G'_{(i-1)}$ (i.e., $w_{G'_{(i-1)}}(u, v)$). (Note: $w_{G'_{(i-1)}}(u, v) = \infty$ if there is no edge (u, v) on $G'_{(i-1)}$).*

(3) ***Operation 3 (Occupant Update):*** *The* occupant *of (u, v) on $G'_{(i)}$ is set to x, where x is the passing vertex along the shortest ternary valley path from u to v on $G'_{(i-1)}$.*

With this above principle, the concept of "occupant" has its physical meaning as follows. Given an edge (u, v) in $G'_{(i)}$, the *occupant* of an edge (u, v) in graph $G'_{(i)}$ is defined to be the (only one) passing vertex of the shortest *ternary* valley path from u to v on $G'_{(i-1)}$. Note that it is possible that an edge (u, v) in $G'_{(i)}$ has an un-assigned occupant (since there is no ternary valley path from u to v).

Note that *UE* (containing the UE spatial graph and the mapping tables) is updated after these operations are performed.

With Principle 1, it is easy to show the correctness of the ternary valley path property (i.e., Property 2) and the shortcut property (i.e., Property 1) by induction.

Theorem 1. UE *satisfies both the ternary valley path property and the shortcut property.*

proof sketch. We first prove by induction a proposition that for each $i \in [1, n]$ and any two distinct vertices u and v in G, the length of the shortest valley path from u to v on the original graph G, which involves only vertices from the set $\{v_1, v_2, ..., v_i\}$ as passing vertices in the original graph G, is equal to $\min\{w, w'\}$, where w is the length of the shortest

ternary valley path from u to v on $G'_{(i)}$ and w' is the weight of (u, v) on $G'_{(i)}$. The correctness for the base case when $i = 1$ clearly holds (due to the shortcut creation and the weight update in Principle 1). Next, we prove by induction from cases 1, 2, ..., i to case $i + 1$.

Consider a pair of vertices u and v and the shortest valley path P from u to v on G. We discuss two cases. Case 1: P involves at least one vertex in $\{v_1, v_2, ..., v_{i+1}\}$ as passing vertices. Assume that the proposition is true in cases 1, 2, ..., i. Let v_k denote the passing vertex on P with the greatest rank value. Note that k is in the range of $[1, 2, ..., i + 1]$. Thus, in $G'_{(i)}$, the shortest valley path from u to v_k and the shortest valley path from v_k to v have the length equal to $\min\{w_1, w'_1\}$ and $\min\{w_2, w'_2\}$, respectively, where w_1 is the length of the shortest ternary valley path from u to v_k on $G'_{(k)}$, w'_1 is the weight of (u, v_k) on $G'_{(k)}$, and w_2 and w'_2 have similar definitions. Consider the ternary valley path (u, v_k, v) on $G'_{(i+1)}$. Its length is $\min\{w_1 + w_2, w'_1 + w_2, w_1 + w'_2, w'_1 + w'_2\} = \min\{w_1, w'_1\} + \min\{w_2, w'_2\}$ because v_k must be contracted via the vertex contraction operation. Since v_k is on P, by proposition, the length of P is equal to the length of (u, v_k, v). Case 2: P doesn't involve any vertex in $\{v_1, v_2, ..., v_{i+1}\}$ as passing vertices. This implies that P is (u, v). Thus, the length of P is equal to the weight of (u, v) on $G'_{(i)}$.

With the proposition and Principle 1, we could easily derive these 2 points: (1) UE satisfies the ternary valley path property and (2) UE satisfies the shortcut property. □

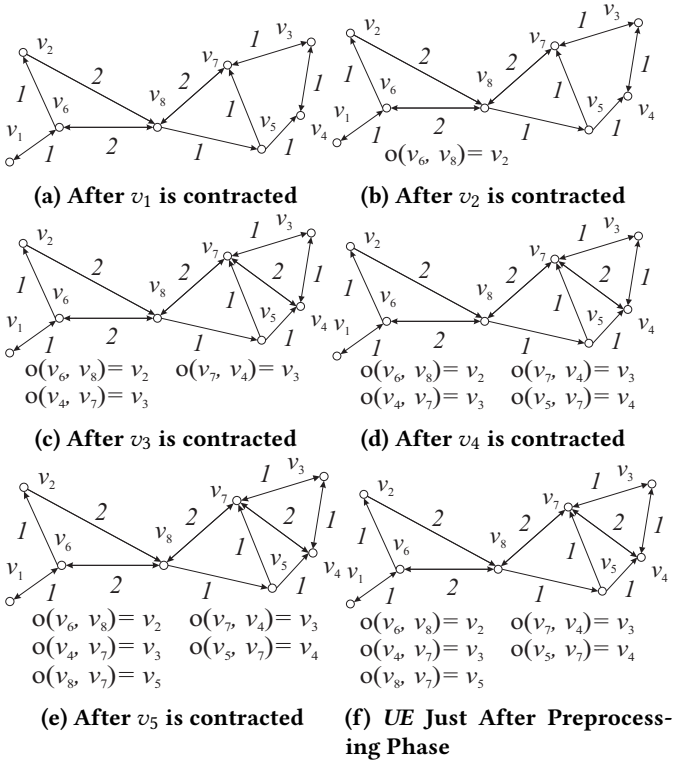
It is not difficult to implement the vertex contraction operation of a vertex according to Principle 1. The following gives an example of this preprocessing phase.

Example 1 (Preprocessing Phase). Consider a permutation $I = (v_1, v_2, ..., v_8)$ of the 8 vertices in Figure 1. Figure 2(a) - (e) shows *UE* after the vertex contractions of the first five vertices and the contractions of v_6, v_7 and v_8 are trivial, and thus, they are not shown in the figure. Initially, G' is set to G. The first step is to contract v_1. Since in G', $N^{in}_{G'}(v_1) = N^{out}_{G'}(v_1) = \{v_6\}$, in this step, we update nothing (since we only process a pair of distinct vertices, where one is from $N^{in}_{G'}(v_1)$ and the other is from $N^{out}_{G'}(v_1)$ according to Principle 1). Figure 2(a) shows *UE* after v_1 is contracted.

The second step is to contract v_2. Since $N^{in}_{G'}(v_2) = \{v_6\}$ and $N^{out}_{G'}(v_2) = \{v_8\}$. Since (v_6, v_8) is an edge in G' and the weight of (v_6, v_8) (i.e., $w_{G'}(v_6, v_8)$) (which is 2) is smaller than $w_{G'}(v_6, v_2) + w_{G'}(v_2, v_8)$ (which is $1 + 2 = 3$), we do not need to update the weight of (v_6, v_8). Since the shortest ternary valley path from v_6 to v_8 on G' is equal to (v_6, v_2, v_8), and $o(v_6, v_8)$ does not exist, we set $o(v_6, v_8)$ to be v_2. Figure 2(b) shows *UE* after v_2 is contracted.

The third step is to contract v_3. Since $N^{in}_{G'}(v_3) = N^{out}_{G'}(v_3) = \{v_4, v_7\}$, we consider two pairs, namely (v_4, v_7) and (v_7, v_4). Since these two pairs (or edges) could not be found in G', we create two edges (v_4, v_7) and (v_7, v_4), and set their weights to $w_{G'}(v_4, v_3) + w_{G'}(v_3, v_7) = 1 + 1 = 2$ and $w_{G'}(v_7, v_3) + w_{G'}(v_3, v_4)\} = 1 + 1 = 2$, respectively. Accordingly, both $o(v_7, v_4)$ and $o(v_4, v_7)$ are set to v_3. Figure 2(c) shows *UE* after v_3 is contracted.

Similarly, we obtain *UE* after v_4 and v_5 are contracted, as shown in Figure 2(d) and Figure 2(e), respectively. The final *UE* is shown in Figure 2(f). □

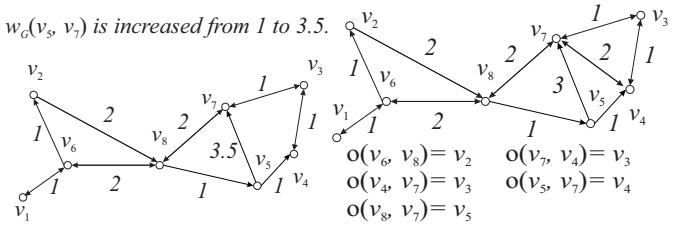


(a) After v_1 is contracted (b) After v_2 is contracted

(c) After v_3 is contracted (d) After v_4 is contracted

(e) After v_5 is contracted (f) *UE* Just After Preprocessing Phase

Figure 2: Illustration of Preprocessing



(a) Spatial Network G with An Edge Weight Update (b) *UE* After Update(G', (v_5, v_7), 3.5)

Figure 3: Illustration of Updating

4.4 Query Phase

In the query phase, given a source vertex s and a destination vertex t, we want to find the shortest distance/path from s to t efficiently based on *UE*. In the following, we describe the shortest distance query and the shortest path query.

Consider the shortest distance query. One straightforward implementation is to adopt a bi-directional Dijkstra's algorithm from both s and t specified in the query. However, it needs to process *a lot of* edges in the UE spatial network.

In the following, we propose a strategy of this implementation so that this implementation could be sped up by just choosing *some* of the edges only for expansion in the UE spatial network, and the shortest distance could be returned. Specifically, these chosen edges are called "upward edges" and "downward edges". We call an edge (u, v) an *upward edge* (resp. *downward edge*) if $R_I(v) > R_I(u)$ (resp. $R_I(v) < R_I(u)$). In our query phase, we call the bi-directional Dijkstra's algorithm in which the forward (resp. backward) Dijkstra's algorithm only expands upward (resp. downward) edges. We call this algorithm, our implementation, as the bi-directional Dijkstra's algorithm with the *direction constraint*.

The remaining question is to show the correctness of this algorithm in this query phase. Before we show the correctness, we present the *ascending-descending property* that is based on the concept of "ascending/descending path". A path is called an *ascending path* (resp. a *descending path*) if each vertex excluding the first vertex has a greater (resp. smaller) rank value than its predecessor along the path. The property is shown as follows.

Observation 1 (Ascending-Descending property). *Consider any two vertices s and t and the vertex v on $s \rightarrowtail_G t$ with the greatest rank value and the UE spatial network G', if s (resp. t) is not v, there exists one s-v (resp. v-t) shortest path on G' that is an 'ascending path' (resp. a 'descending path').*

proof sketch. In a word, due to Property 1, each subpath, which is a valley path, on the s-v and v-t shortest path on G corresponds to a shortcut on G' and the shortcut 'bridges' the two ends of the valley that make one s-v (resp. v-t) shortest path an ascending path (resp. descending path). □

By Theorem 1 and Observation 1, it is easy to show that the bi-directional Dijkstra's algorithm with the direction constraint returns the shortest distance.

Consider the shortest path query from a vertex s to another vertex t on G. Note that the original method described above (i.e., the bi-directional Dijkstra's algorithm with the direction constraint) for the shortest *distance* query on the UE spatial network G' could be applied to find the shortest *path* $\pi_{G'}$ from s to t on G' (not the original graph G). Since we are interested in the shortest path π_G on G (not G'), we should map back the path $\pi_{G'}$ on G' to the shortest path π_G on G with the help of the concept of "real path". Given the shortest path $\pi_{G'}$ from s to t on G', a path π_G is said to be the *real path* of $\pi_{G'}$ on G if (1) π_G is the shortest path from s to t on G, (2) all vertices along $\pi_{G'}$ appears in the same order as the ones along path π_G, and (3) the length of $\pi_{G'}$ (on G') is equal to the length of π_G. In this query phase, we need to consider how to compute the real path on G.

Our algorithm involves the following two steps. The first step is to perform the bi-directional Dijkstra's algorithm from s to t with the direction constraint and obtains the shortest path $\pi_{G'}$ on G'. The second step is to find the real path of $\pi_{G'}$ on the original graph G by (recursively) finding the occupant of each edge (u, v) along $\pi_{G'}$ and inserting it between vertex u and vertex v along the path.

We proceed to present the correctness of our path query algorithm in the following theorem.

Theorem 2. *Our shortest path query algorithm returns an s-t shortest path on the original network G.*

proof sketch. In a word, due to Observation 1, our bidirectional algorithm (which expands upward edges (resp. downward edges) only in the forward (resp. backward) search) finds the shortest path $\pi_{G'}$ in G' and provides the correct distance. Besides, due to our definition of occupant, our query algorithm finds the correct shortest s-t shortest path on G by using $\pi_{G'}$. □

Notice that our distance (path) query algorithm finds the shortest s-t distance (path) no matter what the permutation of vertices is.

Example 2 (Query Processing). Consider the shortest distance query from v_4 to v_5 in Figure 1. It first finds the shortest path from v_4 to v_5 on G' (as shown in Figure 2(f)), which is (v_4, v_7, v_8, v_5), and returns its length. Consider our distance query processing algorithm and it contains a forward (resp. backward) search from v_4 (resp. v_5) that visits upward (resp. downward) edges only and both are performed on G' as shown in Figure 2(f). Consider the forward search from v_4. It first visits the edge (v_4, v_7) since it is an upward edge. Note that (v_4, v_3) will not be visited since it is not an upward edge. Then, it visits the edge (v_7, v_8). The backward search from v_5 is symmetric, which visits one downward edge (v_8, v_5) only. Thus, the two searches finally meet at v_8 and the algorithm finds the path (v_4, v_7, v_8, v_5) and returns its distance 5 as a result of distance query.

Consider the shortest path query from v_4 to v_5. It first obtain the shortest path (v_4, v_7, v_8, v_5) from v_4 to v_5 on G'. Then, it finds its real path by replacing the shortcut (v_4, v_7) with path (v_4, v_3, v_7) (because $o(v_4, v_7) = v_3$) and keeping other edges intact. Thus, the final result of the shortest path query is $(v_4, v_3, v_7, v_8, v_5)$. □

4.5 Update Phase

When there is an update on a weight of an edge on the graph G, we have to update *UE* accordingly so that the shortest

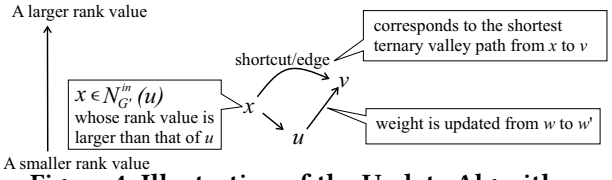


Figure 4: Illustration of the Update Algorithm

distance/path query on the updated *UE* could return the answer correctly.

As we mentioned before, *UE* satisfies the architecture-intact property. In other words, even with a weight update, the architecture of *UE* (i.e., the UE spatial graph) remains unchanged. We are ready to elaborate this point here. To elaborate, let us consider back our oracle constructed based on Principle 1. We know that given a set of weights on all edges in the original graph G, we could construct the UE spatial network G' by introducing shortcuts to the original graph G with the help of Operation 1 of Principle 1. Given another *different* set of weights on all edges in G, we could still construct the *same* UE spatial network G' with the same Operation 1. The reason is that whenever each vertex has its assigned rank value (given in the preprocessing phase), the creation of each shortcut (u, v) in G' depends on whether there *exists* a ternary valley path from u to v on a graph (which relies on the rank values of vertices only) but does not depend on any information about the weights of edges in the graph. Thus, *UE* satisfies the architecture-intact property.

In the update phase, we still follow Principle 1 to maintain our oracle *UE*. Although Principle 1 was originally designed in the preprocessing phase for our oracle construction, it could also be used in maintaining our oracle. As described above, even with a weight update (equivalently, a different weight), Operation 1 of Principle 1 generates the same UE spatial network G'. In the update phase, *UE* also keeps the weight information consistently with the one specified in Operation 2 and the occupant information consistently with the one specified in Operation 3. Thus, only the weight mapping table and the occupant mapping table need to be updated. By following the same principle (Principle 1), *UE* still satisfies the shortcut property and the ternary valley path property, and thus, returns the shortest distance/path correct (as described in the query phase).

Consider that the weight of an edge (u, v) in the original graph G is changed from w to w'. We call algorithm *Update* (shown in Algorithm 1) that takes the UE spatial network G', the edge (u, v) and the new weight w' as input parameters.

We describe Algorithm 1 (i.e., *Update*$(\cdot, \cdot, \cdot)$) as follows. In Line 1, we store the *previous* value of $w_{G'}(u, v)$ in variable w_{before}. In Lines 2-6, we update $w_{G'}(u, v)$ according to 2 different cases. In Line 2, we check whether there exists an occupant $o(u, v)$ for (u, v). If $o(u, v)$ exists, then we compute variable w_o by $w_{G'}(u, o(u, v)) + w_{G'}(o(u, v), v)$ (Line 3) (which corresponds to the length of the shortest ternary valley path). In this case, we update $w_{G'}(u, v)$ with $\min\{w', w_o\}$ (Line 4) (since we want to have a smaller weight value among the newly updated weight w' and the above computed weight w_o). If $o(u, v)$ does not exist, then we update $w_{G'}(u, v)$ with the newly updated weight w' (Line 6). In Line 7, we check whether the previous value of $w_{G'}(u, v)$ is not equal to the updated value of $w_{G'}(u, v)$. If they are not equal, we call *SubUpdate*$(G', (u, v))$ with Algorithm 2 (Line 8).

Next, we describe Algorithm 2 (i.e., *SubUpdate*$(\cdot, \cdot)$) as follows. The major idea is to update the weights of the shortcuts in G' that were originally computed based on the previous value of the weight of (u, v) on G' (i.e., $w_{G'}(u, v)$). There are 2 cases to be considered. The first case is when $R_I(v) > R_I(u)$ (Lines 1-16) and the second case is when $R_I(u) > R_I(v)$ (Line 18). Since these 2 cases are symmetric, we just focus the description on the first case. For illustration, we include Figure 4. In this figure, we place vertices such that a vertex with a larger rank value is placed at an upper level. Thus, this figure shows vertex v is placed higher than vertex u for the first case. In this first case (Lines 1-16), we just need to consider finding back all shortcuts/edges (x, v) in G', where x is a vertex in $N^{in}_{G'}(u)$ whose rank value is larger than that of u, and each of these shortcuts corresponds to the shortest ternary valley paths from a vertex x to the vertex v, (because when each shortcut in G' just described (i.e., (x, v)) was generated (in the pre-processing phase), *UE* considered both edge (x, u) and edge (u, v) together to check whether (x, u, v) is the shortest ternary valley path). In other words, all shortcuts in G' just described (i.e., (x, v)) are those shortcuts whose weight in G' may be updated due to the weight update of (u, v) on G'. Since shortcut (x, v) is an affected shortcut, we have to compute the *updated* weight of (x, v) and the *updated* occupant of (x, v) by considering all possible ternary valley paths from x to v in the form of (x, y, v), where $y \in N^{out}_{G'}(x) \cap N^{in}_{G'}(v)$. In Line 3, we store the *previous* value of $w_{G'}(x, v)$ in variable w_{before}. Lines 4-10 perform the above task of computing the updated weight and the updated occupant. Since we just consider the shortest *ternary* valley path, we do not consider the shortest valley path involving 2 vertices only. Next, we check whether there is an edge (x, v) in the original graph G (which corresponds to the valley path involving 2 vertices) (Line 12). If (x, v) exists, then we update $w_{G'}(x, v)$ with $\min\{w_{G'}(x, v), w_G(x, v)\}$ (Line 13). Finally, in Line 15, we check whether the previous value of $w_{G'}(x, v)$ is not equal to the updated value of $w_{G'}(x, v)$. If they are not equal, we call *SubUpdate*$(G', (x, v))$ with Algorithm 2 (Line 16).

Example 3 (Update Phase). Consider the example of our *UE* in Figure 2(f) just after the pre-processing phase.

Algorithm 1: Update(G',(u, v),w')

Data: The original UE, an edge (u, v) in the original network G whose weight is changed from w to w'
Result: The updated UE

1 $w_{before} \leftarrow w_{G'}(u, v)$;
2 **if** $o(u, v)$ *exists* **then**
3 $\quad w_o \leftarrow w_{G'}(u, o(u, v)) + w_{G'}(o(u, v), v)$;
4 $\quad w_{G'}(u, v) \leftarrow \min\{w', w_o\}$;
5 **else**
6 $\quad w_{G'}(u, v) \leftarrow w'$;
7 **if** $w_{before} \neq w_{G'}(u, v)$ **then**
8 $\quad$ SubUpdate(G', (u, v));

Next, in the original network G, there is a weight update of edge (v_5, v_7) from 1 to 3.5 as shown in Figure 3(a). We call *Update*(G', (v_5, v_7), 3.5) (i.e., Algorithm 1). Variable w_{before} is initialized to $w_{G'}(v_5, v_7)$ (which is 1) (Line 1 of Algorithm 1). Since $o(v_5, v_7)$ exists and is equal to v_4 (Line 2), we compute variable w_o that is equal to $w_{G'}(v_5, v_4) + w_{G'}(v_4, v_7) = 3$ (Line 3). Since the updated weight w' of (v_5, v_7) in G is equal to 3.5, we obtain $w_{G'}(v_5, v_7)$ to be $\min\{w', w_o\} = 3$ (Line 4). Since in G', the original weight value of $w_{G'}(v_5, v_7)$ (i.e., 1) is different from the updated value of $w_{G'}(v_5, v_7)$ (i.e., 3) (Line 7), we call *SubUpdate*(G', (v_5, v_7)) (Line 8). Consider Algorithm 2. Since $R_I(v_7) > R_I(v_5)$ (Line 1 of Algorithm 2), we perform the following steps (Lines 2-16). Since $N^{in}_{G'}(v_5) = \{v_8\}$ and v_8 has a larger rank value than that of v_5 (Line 2), we compute the variable w_{before} as $w_{G'}(v_8, v_7) = 2$ (Line 3). Then, we compute the shortest valley path from v_8 to v_7 via v_5 (i.e., the shortest ternary valley path from v_8 to v_7) (Lines 4-10) and obtain the length of this path (which is $w_{G'}(v_8, v_5) + w_{G'}(v_5, v_7)$) as 2. Since there is an edge (v_8, v_7) in the original G (Line 12), we also compute the shortest valley path from v_8 to v_7 involving 2 vertices only and obtain the length of this path (which is $w_G(v_8, v_7)$) as 2 (Line 13). After Line 13, $w_{G'}(v_8, v_7)$ is still equal to 2 (which is equal to w_{before}). Thus, there is no need to execute the steps in Lines 15-16. The final *UE* after this weight update of (v_5, v_7) in G is shown in Figure 3(b).

□

Theorem 3. *Consider the processing of our update algorithm on the weight change of an edge* (u, v) *on the original network G. (1)* UE *updated by using our update algorithm satisfies both the ternary valley path property and the shortcut property, and (2) the UE updated by using our update algorithm is the same as the UE built from scratch on the most up-to-date road network.*

proof sketch. In a word, the update algorithm is a partial preprocessing algorithm to make *UE* the same as the one built from scrach. □

4.6 Theoretical Analysis

In this part, we analyze the time complexity of the update algorithm as shown in Algorithm 1, which we denote by T_{up}.

Algorithm 2: SubUpdate(G',(u, v))

Data: The original UE, an edge (u, v) in G' with a weight update
Result: The updated UE

1 **if** $R_I(v) > R_I(u)$ **then**
2 $\quad$ **for** *each* x *in* $N^{in}_{G'}(u)$ *whose rank value is larger than that of* u*:* **do**
3 $\quad\quad w_{before} \leftarrow w_{G'}(x, v)$;
4 $\quad\quad$ // Re-compute the shortest ternary valley path from x to v;
5 $\quad\quad w_{G'}(x, v) \leftarrow w_{G'}(x, o(x, v)) + w_{G'}(o(x, v), v)$;
6 $\quad\quad$ **for** *each* y *in* $N^{out}_{G'}(x) \cap N^{in}_{G'}(v)$ *whose rank value is smaller than* x *and* v **do**
7 $\quad\quad\quad w_y \leftarrow w_{G'}(x, y) + w_{G'}(y, v)$;
8 $\quad\quad\quad$ **if** $w_y < w_{G'}(x, v)$ **then**
9 $\quad\quad\quad\quad w_{G'}(x, v) \leftarrow w_y$;
10 $\quad\quad\quad\quad o(x, v) \leftarrow y$;
11 $\quad\quad$ // Consider the shortest valley path from x to v involving 2 vertices only (i.e., path (x, v) with length = $w_G(x, v)$) (if any);
12 $\quad\quad$ **if** *there is an edge* (x, v) *in the original graph* G **then**
13 $\quad\quad\quad w_{G'}(x, v) \leftarrow \min\{w_{G'}(x, v), w_G(x, v)\}$
14 $\quad\quad$ // Update UE again when the weight of (x, v) in G' is updated;
15 $\quad\quad$ **if** $w_{before} \neq w_{G'}(x, v)$ **then**
16 $\quad\quad\quad$ SubUpdate(G', (x, v));
17 **else**
18 $\quad$ (Case: $R_I(u) > R_I(v)$. Details omitted, as it is symmetric to the case of $R_I(v) > R_I(u)$.)

Table 2: Characteristics of Datasets

Name	Region	|V|	|E|	γ
WL	Wales	517,480	1,333,902	3.12
SC	Scotland	906,031	2,308,460	3.02
EN	England	6,339,385	11,505,757	3.09
GB	Great Britain	7,998,285	14,510,323	3.19
ME	Maine	187,315	422,998	≤3
CA	California and Nevada	1,890,815	4,657,742	≤3
C-US	Central US	14,081,816	34,292,496	3.17
US	United States	23,947,347	58,333,344	3.322

Specifically, we show that $T_{up} = O(\log^3(n))$ w.h.p. with five steps, which we sketch as follows.

(1) T_{up} is dominated by the procedure of SubUpdate (line 8 in Algorithm 1), which is shown in Algorithm 2. We denote by $\mathcal{D}$ the maximum in-degree/out-degree of a node in G', i.e., $\mathcal{D} = \max_{v \in V}\{\max\{|N^{in}_{G'}(v)|, |N^{out}_{G'}(v)|\}\}$. Besides, given an edge (u, v) in G, we denote by $\mathcal{F}(u, v)$ the number of edges in G' whose real paths involve (u, v). Let $\mathcal{F}$ denote the maximum $\mathcal{F}(u, v)$ for any edge (u, v) in G, i.e., $\mathcal{F} = \max_{(u,v) \in E} \mathcal{F}(u, v)$. It could be easily verified that T_{up} is bounded by $c \cdot \mathcal{D}^2 \cdot \mathcal{F}$, where c is a constant.

(2) We introduce a concept called *expansion dimension* that captures the growth rate of the number of vertices within a certain number of hops from a vertex in a spatial network. In the existing literature, some similar concepts such as "fractal dimension" [16, 17, 22, 23, 32, 42, 43], "expansion rate" [40] and "KR-dimension" [20, 40] were defined, and were used to measure the growth rate of the number of vertices with a certain *distance* from a vertex in a graph/spatial network. Our expansion dimension is slightly different from these existing concepts since it is based on a certain number of hops but not distance. We empirically compute the expansion dimensions

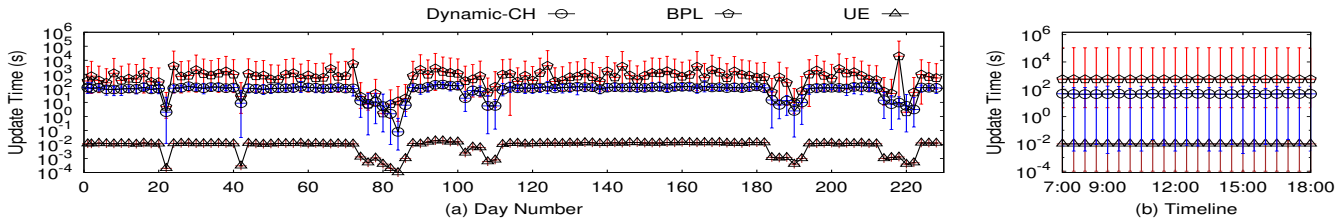


Figure 5: Update Time on GB Dataset

of all datasets we used in our experiments, and the results are presented in Table 2. As could be noticed, the expansion dimensions γ of real spatial networks are usually very small, which are smaller than 3.33 in all cases.

(3) We show that $\mathcal{D}$ is $O(\log n)$ with high probability. Consider a pair of vertices u and v and the path P with the smallest number of hops from u to v. By our preprocessing algorithm, there is an edge (u, v) in G' iff there is a valley path P' in the original network G. By the definition of a valley path, we obtain that the longer the path P', the smaller the probability that (u, v) exists given that the rank values of all vertices are randomly generated. Since the number of vertices on P must be no more than that of P', we derive the upper bound of the probability that (u, v) exists by using the number of vertices on P as a parameter. Together with the conclusion of (2) (i.e., the spatial network has a very small expansion dimension), we could obtain the statistical upper bound of the degree of u.

(4) We show that $\mathcal{F}$ is $O(\log n)$ with high probability. Consider an edge (u, v) on G and a shortcut (x, y) on G'. By our preprocessing algorithm, the real path of (x, y) (on G) contains (u, v) only if the real path is a valley path. Let P' (P'' resp.) denote its sub-path from v to y (from x to u resp.) on G. Consider the path P from v to y on G with the smallest number of hops. Since the number of vertices on P must be at most that of P', we derive the upper bound of the probability that P' is a valley path by using the number of vertices on P as a parameter. Similarly, we could derive the upper bound of the probability that P'' is a valley path. By considering the probabilities of all possible shortcuts containing a given edge (u, v), together with the conclusion of (2) (i.e., the spatial network has a very small expansion dimension), we could obtain the statistical upper bound of $\mathcal{F}$.

(5) With the results in (1), (3), and (4), we conclude that T_{up} is $O(\log^3 n)$ with high probability.

5 EXPERIMENTS

5.1 Experimental Setup

In our experiments, we use a machine with 2.66GHz and 48GB RAM installed with CentOS 5 (x86_64 Edition) linux distribution, where the compiler used is g++4 (GCC) 4.9.2.

Datasets. We use four road networks with real update data (on the edges) in Great Britain, namely WL, SC, GB and EN, where the network data has been provided by geofabrik [1] and the update data has been published by the British government [4]. Specifically, WL, SC and GB datasets contain the update data in 869, 854 and 229 days in 2000-2008, respectively, and EN dataset contains the update data in 31 days of March 2015. In each day, the update data captures how the weights of edges (i.e., amount of time to traverse the corresponding road segments) change every minute from 07:00am to 18:00pm.

Besides, we use four road networks in US, namely, ME, CA, C-US, and US, for scalability tests, where the update data is synthetically generated by using some existing method [25] (which takes a static road network and some traffic of a few road segments such as those in NYC [3] as inputs and generates dynamic traffic on the network). These road networks are well-known benchmarks for the research on static road networks [61, 63]. The characteristics of the datasets are shown in Table 2.

Algorithms. We test our oracle *UE* and seven existing oracles, namely (1) *SILC-Adapt* [50], (2) *PCPD-Adapt* [55], (3) *AH-Adapt* [63], (4) *H2H-Adapt* [46], (5) *Dynamic-CH* [26, 30], (6) *Dynamic-PLL* [11] as shown in Table 1 and (7) *BPL* [34]. Note that we do not test *TVV-Adapt* [8] for the same reason as [63] (i.e., building *TVV-Adapt* needs to materialize all pairwise shortest paths, which is not memory-affordable) and we have tested *BPL* that is not included in Table 1 since it has been shown to perform well empirically though it has no theoretical analysis. We have also tested *BiDijkstra's* Algorithm [27] for the shortest distance and path query. Note that we do not test *Dynamic-HH* since it is dominated by *Dynamic-CH* as verified in [26, 30]. We obtain the C++ implementations of *SILC* [50], *PCPD* [55], *AH* [63], *H2H* [46], *Dynamic-CH* [26, 30] and *Dynamic-PLL* [11], *BPL* [34], from the authors. We implement *UE*, the *BiDijkstra's* algorithm and the updating algorithms of *SILC*, *PCPD*, *AH* and *H2H* (i.e., *SILC-Adapt*, *PCPD-Adapt*, *AH-Adapt* and *H2H-Adapt*) using C++.

Query Generation. Following [61, 63], we generate ten sets of queries $Q_1, Q_2, \ldots\ldots, Q_{10}$ on each dataset as follows. We first estimate the maximum network distance d_{max} between two vertices in the road network. After that, we insert $10,000$

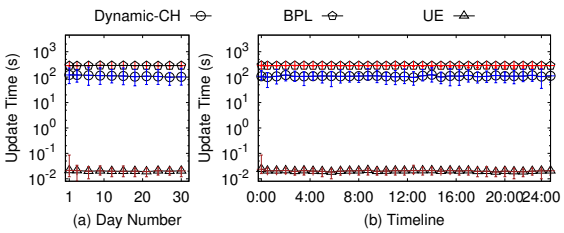


Figure 6: Update Time on EN Dataset

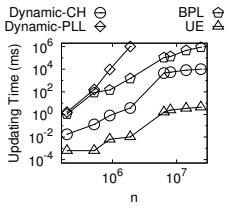


Figure 7: Scalability Test on Updating Time (ms)

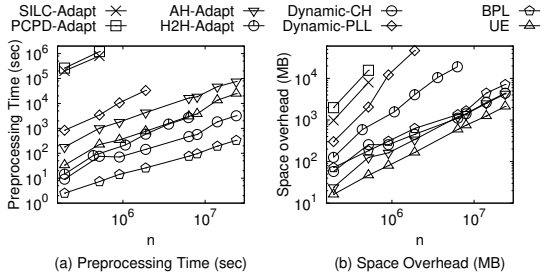


Figure 8: Preprocessing Time and Space

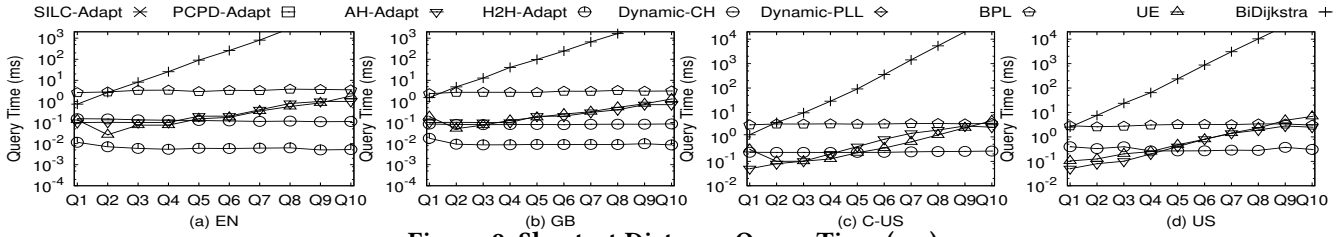


Figure 9: Shortest Distance Query Time (ms)

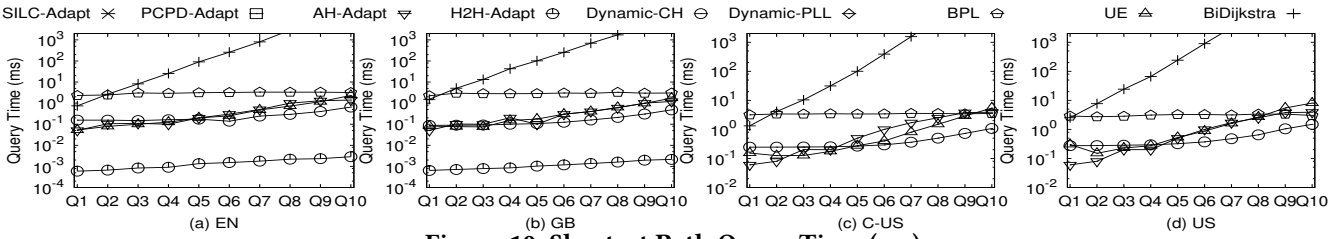


Figure 10: Shortest Path Query Time (ms)

pairs of vertices (s, t) into Q_i, $\forall i \in [1, 10]$ as queries, such that the distance between s and t is in $[2^{i-11}d_{max}, 2^{i-10}d_{max})$. In other words, the network distance between any pair of vertices in Q_i is larger than that in Q_{i-1}.

5.2 Experimental Results

We measure the update time, query time, preprocessing time, and space of all oracles tested. Note that in some cases when an algorithm runs out of the memory budget (48GB), the results are not shown.

(1) Update Time. Since the traffic information of each dynamic network is updated every minute, we regard all the updates in every minute as an updating set and each updating set corresponds to a 1 minute timeslot. For each oracle, we test its update time for all updates in each timeslot of the sampled days from 2000 to 2008 (resp. the 31 days in Mar 2015) on WL, SC and GB (resp. EN). We group all timeslots by day (i.e., all timeslots on the same day (e.g., 1st Mar 2008) are put into the same group) and calculate aggregate information (i.e., average, minimum and maximum update time) of all timeslots in each group. Similarly, we also group all timeslots across different days by the time period (i.e., all timeslots corresponding to the same time period (e.g., 7:00 - 7:01am) are put in the same group) and calculate the aggregate information (i.e., average, minimum and maximum update time) of all timeslots in each group. We conduct experiments on WL, SC, GB and EN. For the sake of space, we only show the results on GB and EN. The reason is that the results on WL and SC are similar to those on GB. The results on GB and EN are shown in Figures 5-6, respectively, where Figures 5-6(a) show the results based on grouping of days and Figures 5-6(b) show the results based on grouping of time periods. In each figure, we adopt a vertical bar to show the aggregate information (i.e., average, minimum and maximum update time) of each timeslot in a group. The lower and upper bounds of the bar corresponds to the minimum and maximum values, respectively. The symbol (e.g., circle, pentagon and triangle) in each bar shows the average value in the group. The results of *SILC-Adapt*, *PCPD-Adapt*, *AH-Adapt* and *H2H-Adapt* are not shown since they would take

more than 1 week to finish updating for a single timeslot. The results of *Dynamic-PLL* are not shown since it would run out of our memory budget.

According to these results, *UE* could be updated in a neglectable amount of time. The update time of *Dynamic-CH*, *Dynamic-PLL* and *BPL* is several orders of magnitude larger than ours on WL and SC. Besides, on GB and EN, the update time of *BPL*and *CH* both exceeds 1 minute for most updating sets, which means they could not meet the hard requirement of the updating in practice. On GB, the update time of each oracle fluctuates from day to day (Figure 5(a)) but keeps almost steady in different time periods (Figure 5(b)). The reason is that the number of edges to be updated in each updating set (corresponding to 1min timeslot) varies a lot in different days but keeps almost intact in different time periods. On EN, the update time of each oracle keeps almost steady (Figure 6(a) and (b)) since on EN, the number of edges to be updated in each updating set (corresponding to 1min timeslot) does not vary too much in all timeslots.

We also do a scalability test on the update time based on all the eight datasets including those with synthetic update data so that datasets with different sizes (denoted by n) are used. The results are presented in Figure 7. According to this figure, our *UE* oracle is scalable and outperforms the baselines by several orders of magnitude.

Although only our *UE* oracle could handle the update operation efficiently (i.e., the data update time is within the processing time for update), in the following, for the sake of interest, we also compare *UE* with other existing oracles described before in other measurements (i.e., query time, preprocessing time and space).

(2) Query Time. Figure 9 (resp. Figure 10) presents the shortest distance (resp. path) query time of the tested oracles. For the sake of the limited space, we only show the results on the 4 largest datasets as shown in Table 2. As these results generally show, (1) for queries with relatively smaller shortest distances (e.g., Q1-5), all algorithms except for *BiDijkstra* and *BPL* perform quite well and have comparable query time; (2) for other queries (e.g., Q6-10), *H2H-Adapt* works the best in most cases (note that *H2H-Adapt* takes more than one week to finish updating a single time slot in our previous experiment); and (3) our *UE* oracle works fairly well for all queries on all datasets (e.g., the query time is at most 10ms).

(3) Preprocessing Time and Space. Figures 8(a) and (b) show the preprocessing time and the space of the tested oracles on the networks (with varying sizes). As shown in Figure 8(a), (1) *PCPD-Adapt* and *SILC-Adapt* have their preprocessing time multiple orders of magnitude larger than that of other oracles and thus cannot scale up to large or even moderate size datasets; (2) *Dynamic-PLL* and *H2H-Adapt* have a smaller preprocessing time compared with *PCPD-Adapt* and *SILC-Adapt* but still cannot scale up to large datasets; and (3) our *UE* oracle has its preprocessing time the third smallest and can scale up to the largest dataset in our experiments. As shown in Figure 8(b), *PCPD-Adapt*, *SILC-Adapt*, and *Dynamic-PLL* have their space 1-3 orders of magnitude larger than that of other algorithms and cannot scale up to large datasets and all other oracles have their space comparable but our *UE* oracle has clearly the smallest space.

(4) Experimental Summary. Firstly, our *UE* oracle is superior over all existing ones for applications, where it is required that update needs to be handled in time (simply because our *UE* can handle the updates very quickly while none of the existing oracles can handle the update before the next batch of update arrives). Secondly, our *UE* oracle allows for very efficient query processing (e.g., less than 10ms for all queries on all datasets tested in this paper), needs quite affordable processing time (e.g., a few hours for the largest dataset), and costs the smallest space. Thirdly, *PCPD-Adapt*, *SILC-Adapt*, *Dynamic-PLL* and *H2H-Adapt* have serious problem on their space that are 2-3 orders of magnitude larger than others and could not scale up to sizable datasets. Besides, each baseline has an update time more than 2 orders of magnitude larger than that of *UE*.

6 CONCLUSION

The paper studies the shortest distance and path queries on the dynamic road network and develops a very efficient oracle for this problem. This oracle has decent theoretical bounds on the preprocessing time, space, query time and update time. Remarkably, it enjoys an $O(n \log^2 n)$ preprocessing time and an $O(\log^3 n)$ update time w.h.p.. However, the existing oracles all have a worse-than $O(n^{1.5})$ preprocessing time and a worse-than $O(n^{1.5})$ update time, which shows that they are not scalable. Our extensive empirical study verifies that the existing oracles sustain the short updating time in practice. Besides, our oracle significantly outperforms the existing works in terms of the update time. The query time, preprocessing time and space of our oracle is comparable with the best existing work.

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