COMP 4971 – Independent Study (Fall 2018/19)

Optimization of Bollinger Bands on Trading Common Stock Market Indices

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1. Introduction

1.1. Bollinger Bands

Bollinger Bands, propounded by John Bollinger, are a common technical analysis tool defined as a pair of *k*-standard deviation (SD) bands above and below *n*-day moving average (MA) of a financial instrument's closing price (Bhandari, 2016). While MA highlights long-term pricing trend, SD provides measure of volatility in the investigated time series. The combination of MA and SD aims to set a relative benchmark for price fluctuation based on their statistical meaning. Outliers deviated from the bands are identified as signs of trend reversal, suggesting potential trading opportunities. Although it is debatable whether statistical theory of SD still holds for non-normally distributed data of daily price, previous study contended that Bollinger Bands can encapsulate a consistent proportion of historical price (Rooke, 2010), ensuring its reliable ability to capture trend movement. Bollinger bands are typically constructed from 20-day simple moving average (SMA) and 2 SDs of closing prices in that 20 days, but these settings may not be the universal solution for every financial instrument. It is at investors' expense to trade with an unoptimized tool.

The aim of this study is to develop optimization method for the two parameters of Bollinger Bands, i.e. *n* for time frame and *k* for SD multiplier, and evaluate the performance of suggested strategies on historical data of 3 common stock market indices in term of their excess annual return in 3 years investment.

1.2. Common Stock Market Indices to be Analysed

This study selected 3 indices from different stock markets:

1. Hang Sang Index in Hong Kong
2. Standard & Poor's 500 (S&P 500) in the United States
3. Nikkei 225 in Japan

These indices are internationally recognized as representatives of market performance in their corresponding regions. Trend analysis on the indices may highlight overall market strengths and weaknesses, providing insights for particular investments. Moreover, financial instruments trading on the performance of these indices are available in the market, e.g. exchange-traded funds (ETFs), index futures and options, so the insights from the study may be directly applied to trading these instruments.

2. Source of Data and Software Framework

All historical data of listed indices were retrieved from Yahoo Finance by open source python libraries *pandas* and *pandas-datareader*. All source codes for this study were developed on a program *Stoxy*, kindly provided by Prof. David Rossiter. This program was used mainly for visualizing customized Bollinger Bands on daily closing price charts as well as heatmaps for reporting performance of Bollinger Bands in different settings, with the help of another open source python library *matplotlib*.

3. Algorithm Development

The following paragraphs illustrate the development of the Bollinger Bands trading algorithm: formulating an efficient function of Bollinger Bands, extending the idea to capture more potential returns, and describing the flow of the program.

3.1. Moving Average and Moving Standard Deviation Functions

Bollinger Bands are constructed based on MA and its SD over successive time frame. Explicitly, the value of standard *n*-days Bollinger Bands with *k* SD on day *(d+n-1)* can be expressed by following equations (Bhandari, 2016):

$$\text{Upper Bollinger Band}_{day\ (d+n-1)} = \text{SMA}(n, d+n-1) + k \times \text{SD}(n, d+n-1) \quad (1)$$

$$\text{Lower Bollinger Band}_{day\ (d+n-1)} = \text{SMA}(n, d+n-1) - k \times \text{SD}(n, d+n-1) \quad (2)$$

The first term (SMA function) represents the reference level of the smoothened trend, while the second term (SD multiplied with a constant) defines the allowance range of price fluctuation said to be 'within the current trend'. Equations (1) and (2) show that Bollinger Bands are plotted 'symmetrically' above and below the selected SMA line since they have the same distance *k* SD from it. The idea of 'symmetric' will be further discussed in Section 3.3. Before calculation of Bollinger Bands, n-day SMA and its corresponding SD must be calculated, which are:

$$\text{SMA}(n, d+n-1) = \sum_{i=0}^{n-1} \frac{P_{d+i}}{n} \quad (3)$$

$$\text{SD}(n, d+n-1) = \sqrt{\frac{\sum_{i=0}^{n-1} (P_{d+i} - \text{SMA}(n, d+n-1))^2}{n-1}} \quad (4)$$

P_{d+i} denotes the price of the financial instrument on day *(d+i)*. SD is corrected to degree of freedom *(n-1)* as it is calculated based on historical sample data (Berk and DeMarzo, 2016).

To compute successive values in shorter runtime, these standard statistical equations (3) and (4) were modified to function in a moving time frame. The following recursive equations can update the stored results to a new time frame by adding the new datum P_{d+n} and removing the oldest datum P_d simultaneously:

$$\mathrm{SMA}(n, d+n) = \mathrm{SMA}(n, d+n-1) + \frac{P_{d+n}}{n} - \frac{P_d}{n} \tag{5}$$

$$\mathrm{SD}(n, d+n) = \sqrt{(\,\mathrm{SD}(n, d+n-1)\,)^2 + \frac{{P_{d+n}}^2 - {P_d}^2}{n-1} - \frac{2(P_{d+n}-P_d)\times \mathrm{SMA}(n, d+n-1)}{n-1} - \frac{(P_{d+n}-P_d)^2}{(n-1)n}} \tag{6}$$

Figures 1 shows the comparison of Bollinger Bands plotted by an external source and *Stoxy* in the same time interval to reflect the accuracy of the functions developed.

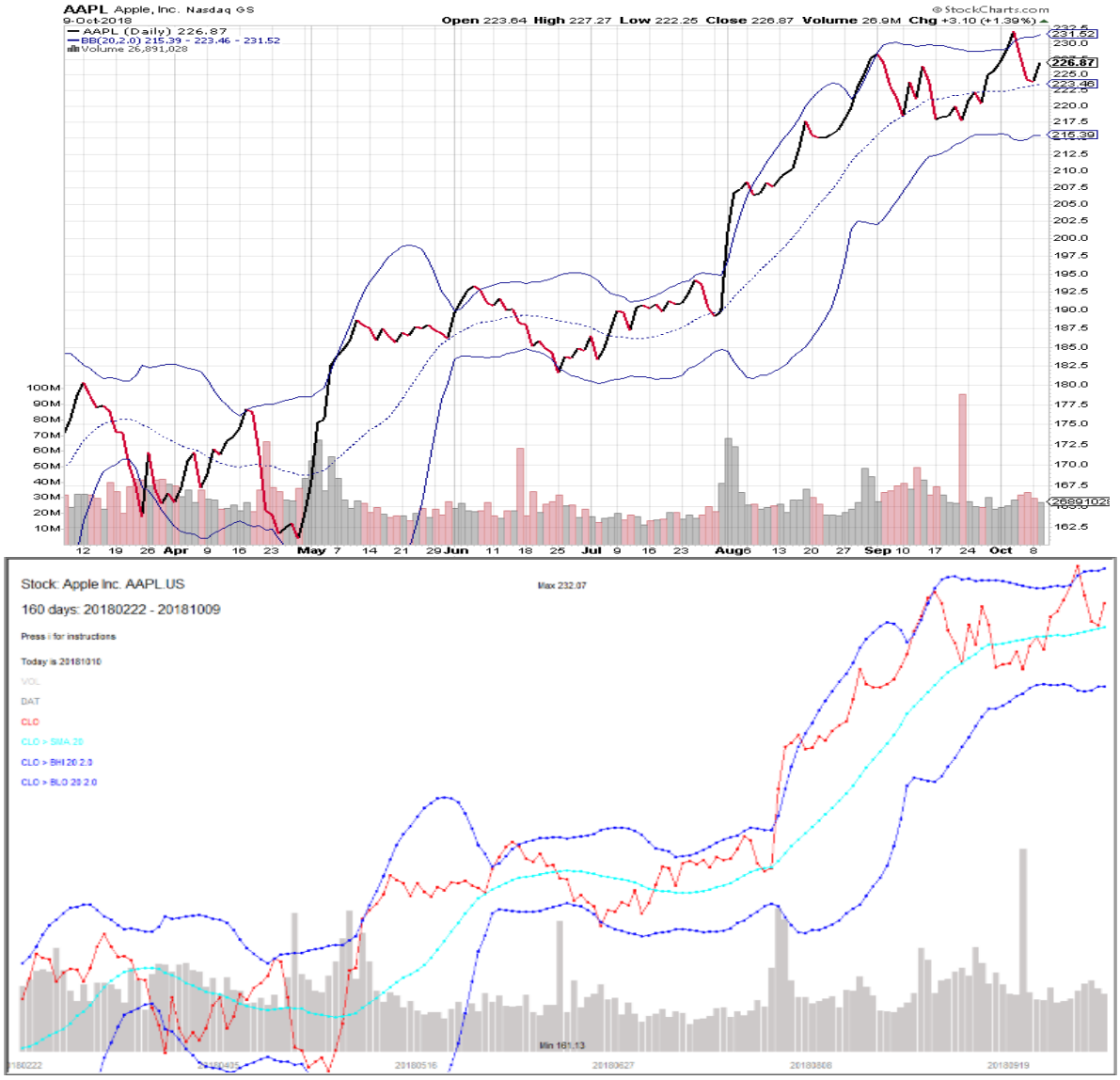

Figure 1 Bollinger Bands plotted by StockChart.com (top) and Stoxy (bottom) for Apple Inc. stock price from 9th March 2018 to 9th October 2018

3.2. Exponentially Weighted Bollinger Bands

Bollinger Bands typically use SMA as its reference line to seek for breakout of current trend, but this idea is not confined to this averaging method. Exponential moving average (EMA), for example, can be set as the reference instead. Data of closing price are multiplied with a weighting factor, which decrease exponentially from the most recent datum to the first existing datum, when the moving average is calculated (Finch, 2009). To be associated with EMA, the SD involved in this modified Bollinger Bands is also adjusted to be exponentially weighted accordingly.

The exponentially weighted variance and SD is denoted as EWVar and EWSD in this study. Exponentially Weighted Bollinger Bands with *n* days as time frame and *k* as SD multiplier for the prices of the financial instrument *P* can be calculated by the following pseudocodes:

$\alpha = \frac{2}{n+1}$

$EMA = P[0]$

$EWVar = 0$

For item *i* in *P* after *P*[0]:

$\delta = P[i] - EMA$

$EMA = EMA + \alpha \cdot \delta$

$EWVar = (1 - \alpha)(EWVar + \alpha \cdot \delta^2)$

$EWSD = \sqrt{EWVar}$

$Upper\ band = EMA + k \cdot EWSD$

$Lower\ band = EMA - k \cdot EWSD$

Since recent data are weighed more heavily than legacy data, EMA and its resulting EW Bollinger Bands can follow the trend movement more tightly. Figure 2 shows narrowing of the bands (indicating a reduction of volatility) happened faster and more drastically for the EW one than the standard one, suggesting the potential of EW Bollinger Bands to capture trend changes with less delay.

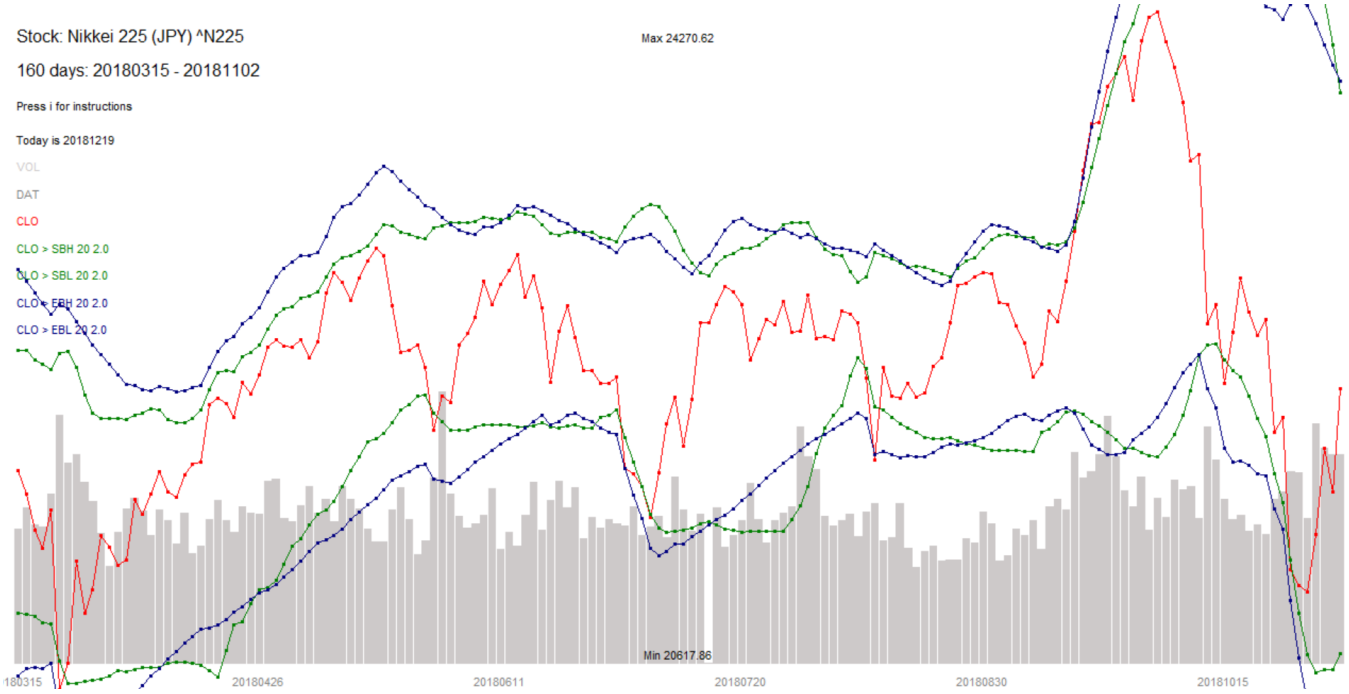

Figure 2 Standard Bollinger Bands (green) and EW Bollinger Bands (blue) Plotted for Nikkei 225 from 15th March 2018 to 2nd November 2018

The algorithm suggests trading opportunities when latest trend breaks out of or return to the area encapsulated by Bollinger Bands. The financial instrument is bought (sold) when the current price first moves outside of the upper (lower) band, expecting the suspected upward (downward) trend to continue. When the outlying price return to the area inside the Bollinger Bands from the upper (lower) side, the financial instrument is sold (bought) as the confident upward (downward) trend is over. Also, a possible trend reversal may occur when investors recognize the overbought (oversold) situation. Therefore, the zone above the

upper band is the period where only the financial instrument is held, and the zone below the lower band is where only cash is held, illustrated by Figure 3.

Figure 3 Notation of Buy and Sell Signals on Bollinger Bands

Since the stock is bought as much as possible after the price movement exiting from the 'cash only' zone, there is no extra buy signals when the trend enters the 'stock only' zone in the first two buy-sell phases shown in Figure 3. After Phase 2, the price level does not touch the lower band, which is supposed to trigger buy activities at relatively low price. Therefore, it is necessary to set remedial buy signal when the price trend enters the 'stock only' zone again in Phase 3 to capture the profitable upward movement. In each intersection, only one band is considered so no signal conflicts can occur.

3.3. Asymmetric Bollinger Bands

Bullish and bearish trends may not perform in the same patterns in terms of return and volatility, implying that the trend following strategies for upward and downward trends may be different. Bollinger Bands have the potential to address to this issue since individual bands can provide responds to the changing trend separately. When there is a strong upward (downward) trend, the price is very likely to move to a relatively high (low) level, which is indicated by the breakout from the upper (lower) bound of Bollinger Bands.

Therefore, the upper bands can be responsible to tracing the upward price movement, while the lower band adopt the role to follow the downward trend.

Since the bands are used to follow different patterns, this study suggested using different parameters for optimization of upper and lower bands, which makes Bollinger Bands asymmetrically plotted away from the referencing MA. This may relax the restrictions of standard Bollinger Bands, promoting its ability to provide immediate signals to different trend motions. Figure 4 illustrates the mechanism of asymmetric Bollinger Bands.

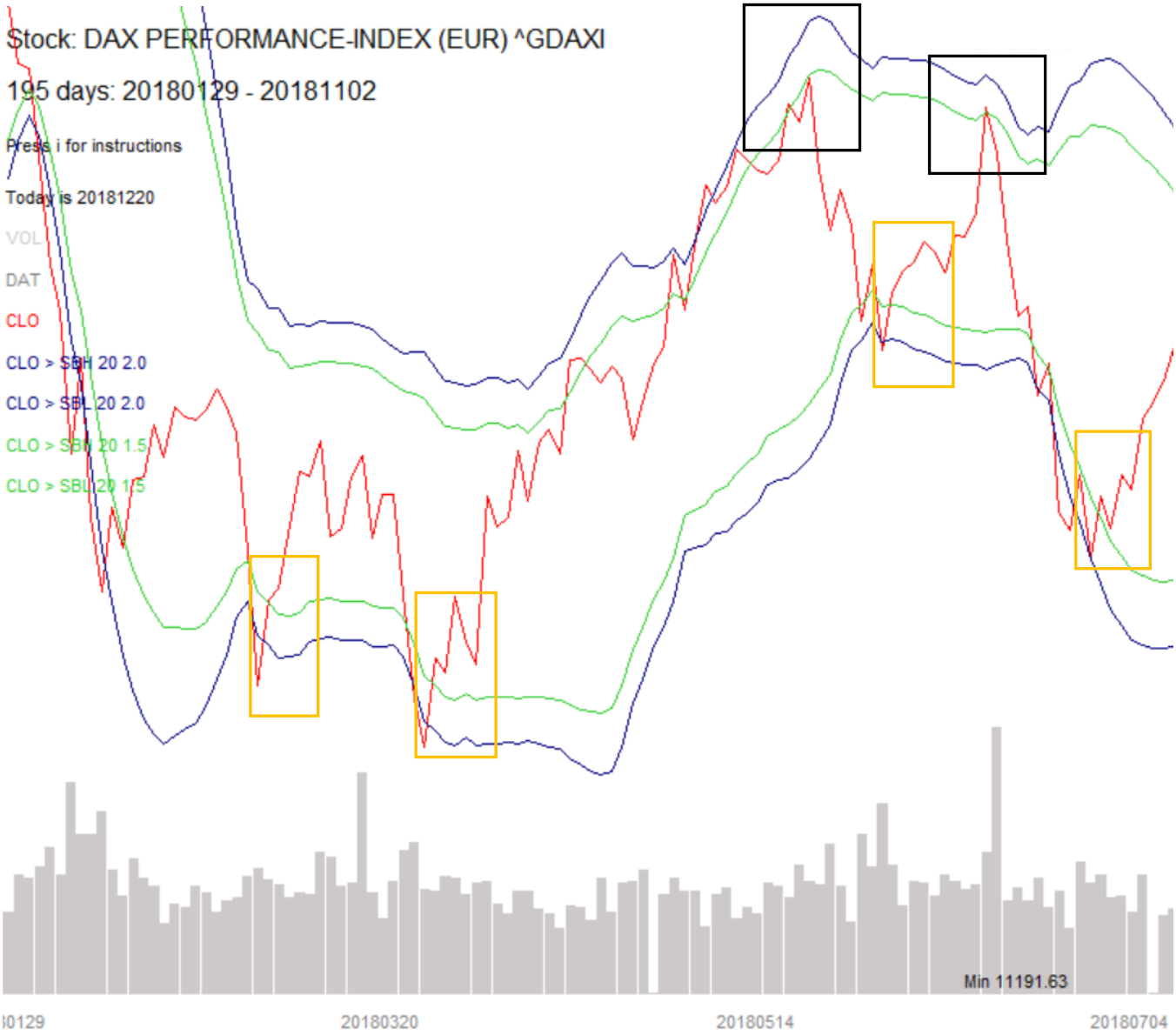

Figure 4 Bollinger Bands with 1.5 SD (green) and 2 SD (blue) away from 20-day SMA

In the orange zones, the closing price intersects the blue lower band at lower and earlier points than the green lower band, signalling the recovery from the troughs sooner. Investment can be triggered quicker at a lower price to increase overall return with this 2SD lower band. However, the blue upper band fails to detect the peaks in the black zones, while the green upper band is capable to signal the immediate sell signals. The 1.5 SD upper band thus performs better than the 2 SD upper band in upward trend following. By combining performance of the 2 SD lower band and 1.5 SD upper band, the asymmetric Bollinger Bands can trace trend reversals during this period more accurately and generate higher return.

The trading performance of Bollinger Bands as well as the performance of individual bands can be reported by heatmaps, in which the return generated by the strategies is scaled as the 'temperature'. A typical example is shown in Figure 5.

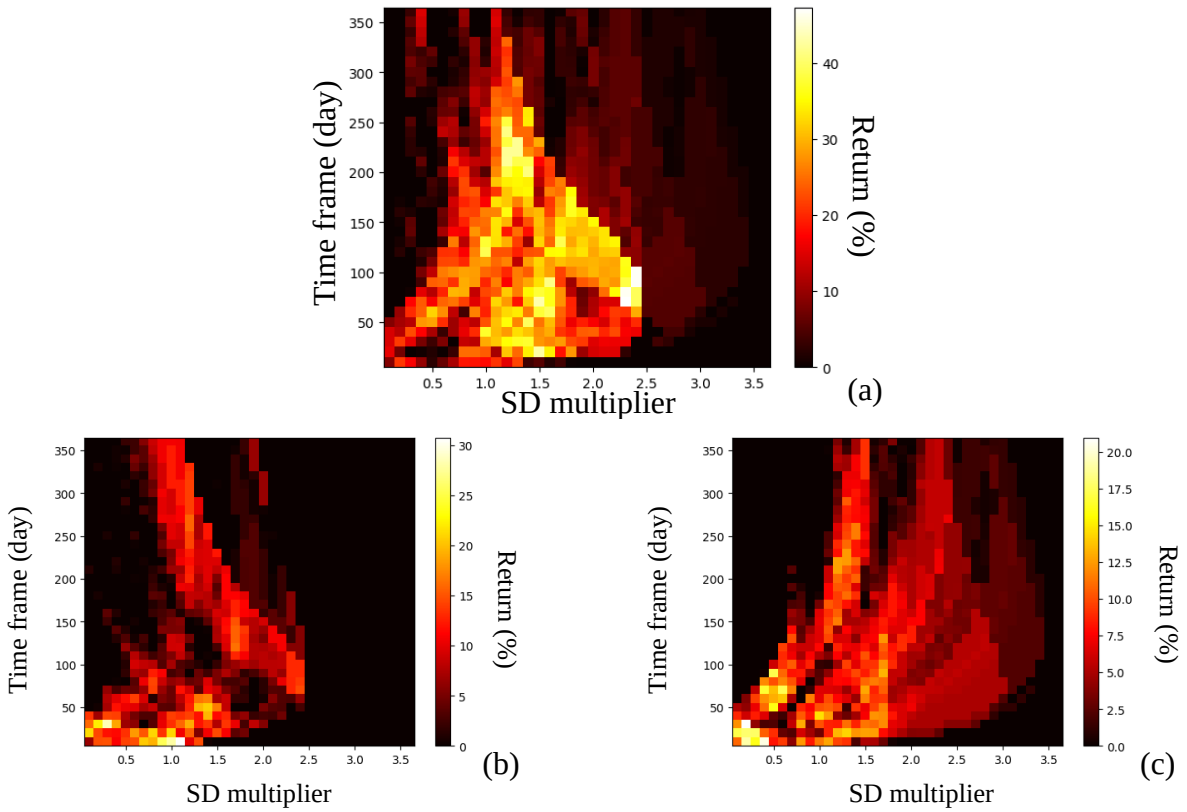

Figure 5 Heatmaps of Return by Trading with Bollinger Bands (a), Lower Band (b) and Upper Band (c) with Varying Time Frame and SD Multiplier

Fan shape patterns are generated in the heatmaps, hypothesized as profitable regions formed by pairs of buy and sell activities (phases described in Figure 3). The fan-shaped regions on the heatmap of Bollinger Bands can also be identified in either one of the subplots (although the return, i.e. brightness of spots, differs). This suggested that return generated by Bollinger Bands may be considered as superposition of returns produced by upper band and lower band separately.

This finding can simplify the optimization method for asymmetric Bollinger Bands. Upper band and lower band generating the highest return individually are selected, and they are expected to provide even higher return cooperatively when the peaks and toughs are identified earlier than standard Bollinger Bands.

3.4. Settings, Assumptions and Flow of the Trading Algorithm

The performance of standard, EW and asymmetric Bollinger Bands was evaluated by their excess return in the simulation with historical data of indices. Excess return was defined as annual return (geometric mean of percentage increase) generated by the trading strategy, subtracted by the annual return generated by buying and holding the financial instrument from the starting date of trading test till the end (denoted as ‘natural growth’ of the instrument hereafter).

The trading test had following settings:

1. Initial budget is 1 million in the same currency of the investigated index.
2. An ETF perfectly following the investigated index is invested with the fund price to index point ratio be 1 dollar to 1 point.

As percentage increase of assets was considered in the trading test, changes of the default values should not alter the results relatively.

However, this study depended on some assumptions:

1. Only two assets were considered, namely cash and the investigated ETF.
2. All trading activities were performed at the adjusted closing price at that day (retrieved from Yahoo finance).
3. Volume of each trading activities were unlimited.
4. No costs were incurred in all trading activities.

Although costs of investment were neglected in the trading tests, total numbers of trades made in each simulation were noted.

The algorithm first simulated the trading on the investigated index ETF in *m* successive years with standard and exponentially weighted Bollinger Bands. Six bands with the highest annual return among the following categories were selected:

A. Standard Bollinger Bands
B. Upper standard band
C. Lower standard band
D. Exponentially weighted Bollinger Bands
E. Upper EW band
F. Lower EW band

Then, the trading performance of these selected bands in the next *n* successive years were tested with the following combinations:

1. Standard Bollinger Bands (A)
2. Exponentially weighted Bollinger Bands (D)
3. Upper standard band (B) + Lower standard band (C)
4. Upper standard band (B) + Lower EW band (F)
5. Upper EW band (E) + Lower standard band (C)
6. Upper EW band (E) + Lower EW band (F)

The combination with the highest excess return was reported as the optimized solution of trading the investigated index in that *m*+*n* years. If one trending following method was consistently selected as the optimized solutions, it would be concluded as the most suitable strategy for technical analysis in that stock market index.

4. Results and Discussions

4.1. Performance Evaluation of the Algorithm in Short, Medium, and Long Terms

This study investigated the performance of the algorithm based on historical data of the 3 selected indices from beginning of 2006 to beginning of 2018. The testing data were split into 10 successive sets with time frames of 3 years long, e.g. from beginning of 2006 to beginning of 2009 and from beginning of 2007 to beginning of 2010.

The algorithm first simulated trading with Bollinger Bands of:

1. Time frame ranging from 10 days to 360 days (at intervals of 10 days) and
2. SD multiplier ranging from 0.1 to 3.6 (at intervals of 0.1)
3. With training data from 3/ 6/ 9 years before each testing data sets.

The size of the training data defines the period of investigation, i.e. training with 3 years of data is referred as 'short term', 6 years of data as 'medium term' and 9 years of data as 'long term'. Bollinger Bands with the highest return in the training simulation were then selected and evaluated with the testing data sets as methods described in Section 3.4.

4.1.1. Results of Trading on Hang Sang Index

Table 1 Statistics of Bollinger Bands Suggested for Trading Hang Sang Index from 2006 to 2018

	Short Term	Medium Term	Long Term
Number of asymmetric bands	7	9	5
Number of standard upper bands	5	6	8
Average time frame of standard upper bands (days)	158	163	59
Average SD multiplier of standard upper bands	0.64	0.33	0.45
Number of EW upper bands	5	4	2
Average time frame of EW upper bands (days)	84	73	190
Average SD multiplier of EW upper bands	0.7	0.25	1.15
Number of standard lower bands	5	5	7
Average time frame of standard lower bands (days)	188	112	117
Average SD multiplier of standard lower bands	1.7	1.2	0.67
Number of EW lower bands	5	5	3
Average time frame of EW lower bands (days)	150	116	127
Average SD multiplier of EW lower bands	0.96	0.62	1
Average Excess Return (%)	3.943	4.751	5.572

Table 1 shows that asymmetric Bollinger Bands were frequently implemented to trace the trend of Hang Sang Index, indicating the importance to separate trend following strategies for bullish and bearish market performance. Upper bands tend to have shorter time frames and smaller SD multipliers than lower bands in both standard type and exponentially

weighted type. This may suggest that upward price movement happened more suddenly and required early detection of the trend to be profitable.

Usage of standard bands were about the same as the usage of exponentially weighted bands, except in long term investigation where standard upper and lower bands were more preferred. The standard bands suggested in long term investigation usually have time frame shorter than 60 days and SD multiplier smaller than 1 (Table 4 in Appendices), which were optimized for tracing short term price fluctuation according to the theory. This also coincided with the frequent trades noted, when the algorithm signalled buy and sell activities for minor price peaks and toughs happened weekly. However, the long term investigation also suggested EW Bollinger Bands with time frame of 320 days and SD multiplier of 2.2, which was optimized for tracing long term financial crashes and recovery. Figure 9 in Appendices shows that these EW bands were touched only when stock market crises started and ended.

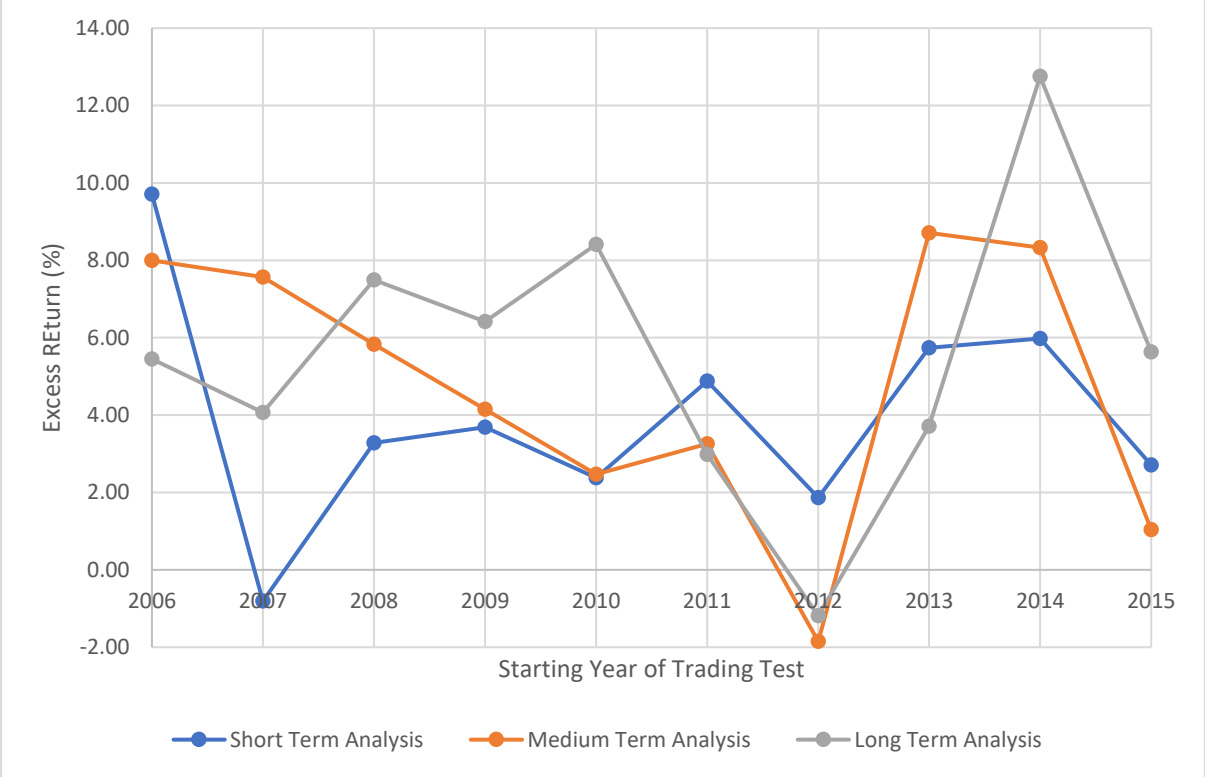

Figure 6 Excess Return for 3-Years Trading Test on Hang Sang Index

Figure 6 shows that the trades suggested by the algorithm can largely generate positive excess return on Hang Sang Index, i.e. perform better than overall stock market performance. Among the three period of investigation, long term analysis performed the best for half of the trading tests. Furthermore, these superior results of long term investigation were using the same strategy, i.e. standard Bollinger Bands with time frame of 10 ~ 60 days and SD multiplier of 0.1 ~ 0.9 (Table 4 in Appendices). Since it was suggested by the algorithm consistently, it will probably be an adequate solution to produce promising return if the current trend continues. The disadvantage of using such narrow Bollinger Bands is the relatively high transaction costs due to frequent trades.

For the trading test starting from 2012, both medium and long term investigation method failed to generate positive excess return (Figure 6), suggesting their worse performance than natural growth of the index. From 2012 to 2015, Hang Sang Index had a steady growth after recovery from the fear of 2011 United States debt-ceiling crisis (yellow zone of Figure 9 in Appendices). However, the algorithm of Bollinger Bands, trained with 6 or 9 years of data before 2012, has adapted to major stock crashes, e.g. 2003 SARS crisis and 2007-2008 global financial crisis. The technical analysis optimized for adverse market environment could not capture minor fluctuations in a steady upward trend, and thus generate returns lower than natural growth of the index. This phenomenon is more significant in the results for S&P500.

In conclusion, asymmetric Bollinger Bands were suggested with upper band having shorter time frames and smaller SD multipliers. While 10 ~ 60-day standard Bollinger Bands with 0.1 ~ 0.9 SD were preferred for capturing short term trading opportunities, EW Bollinger Bands with time frame of 320 days and SD multiplier of 2.2 may be referred to caution against possible market crashes.

4.1.2. Results of Trading on S&P 500

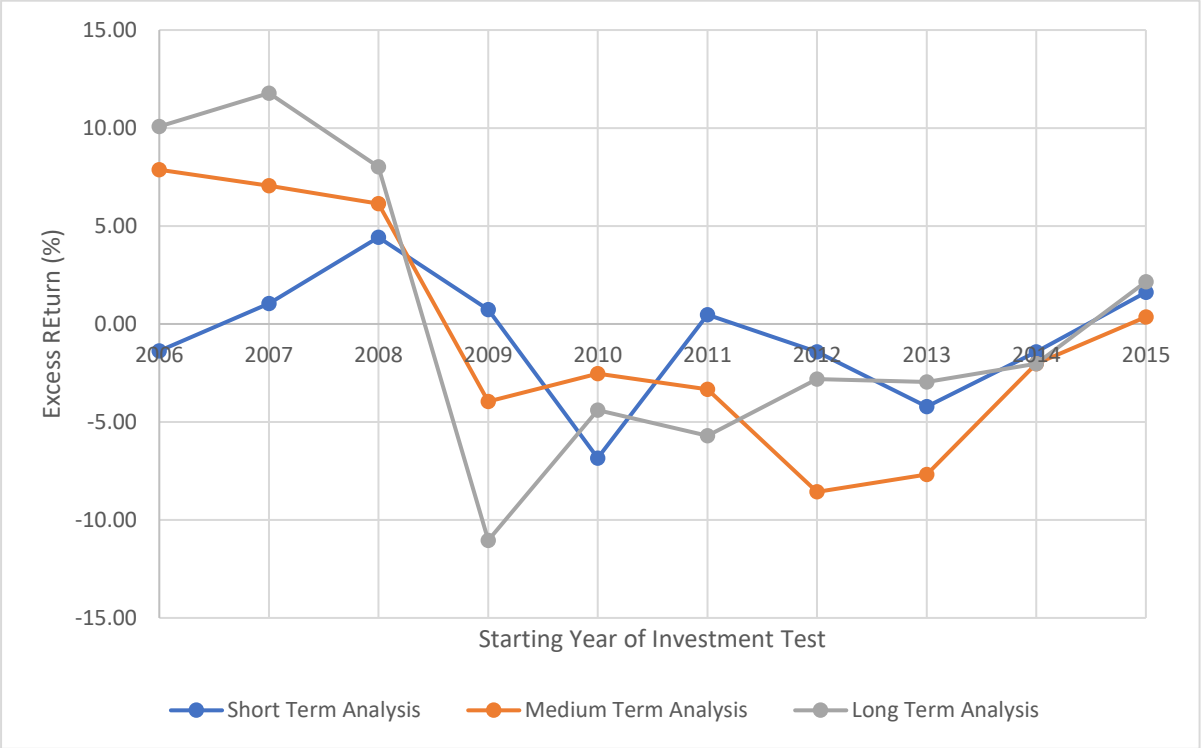

Figure 7 Excess Return for 3-Years Trading Test on S&P 500

Figure 7 shows that Bollinger Band performed worse than natural growth of S&P 500 after 2009, indicated by the excess returns below or close to 0. The failure of the algorithm was addressed by analysing the properties of historical trends.

For years before 2009, the positive excess returns shown were exaggerated due to massive decline of the index points (large magnitude of negative natural growth) during 2007-2008 global financial crisis. Bollinger bands derived from short and medium analysis managed to maintain 0 growth during the adverse situation, while the bands derived from long term investigation produced about 4% annual return (Figure 12 in Appendices). The 9 years of historical data used for training the algorithm included the whole development of 2002 stock market downturn in the United States, so the Bollinger Bands performing well in the downturn were also capable of resisting market crash in 2007-2008 and generate considerable return when majority of the stock market suffered from the recession.

However, the stock market quickly recovered from the recession since 2009, and grew more rapidly than the trend before the crisis (Figure 11 in Appendices). Trends reversals between crisis observed in previous historical data could not assist the prediction of this drastic uptrend, so Bollinger Bands suggested failed to produce significant positive excess return. When the time frame moved closer to recent data and the algorithm adapt to the situation with the new training data, it faced difficulty to search for a relatively low price to signal the initial buying action in such strong uptrend. Cash which brought no return was kept during the delay from the start of trading tests to the first buying signal, so overall annual return of the strategy was lower than the natural growth of index over the period.

In conclusion, Bollinger Bands is not an effective technical analysis tools to handle unpredictable trends, and strong trends with few trend reversals to trigger trades.

4.1.3. Results of Trading on Nikkei 225

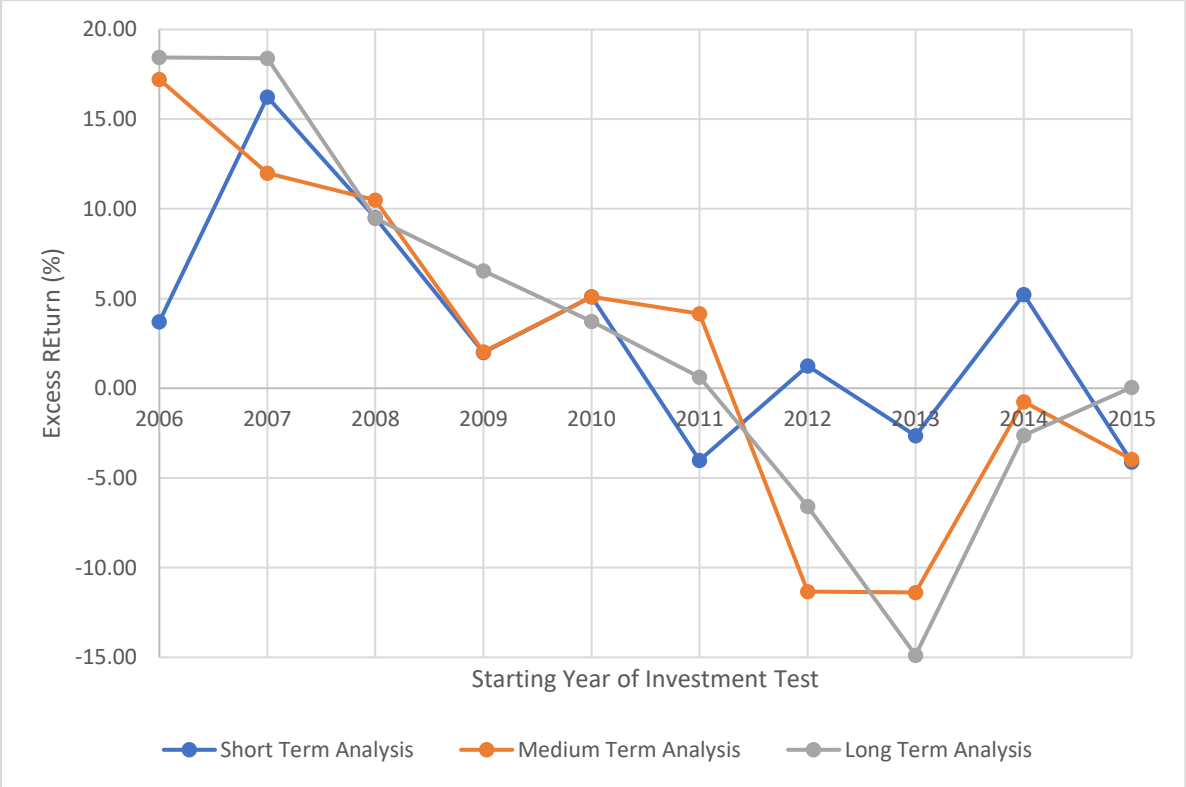

Figure 8 Excess Return for 3-Years Trading Test on Nikkei 225

The performance of Bollinger Bands on Nikkei 225 also showed the exaggeration of positive excess return before 2009 and the negative excess return after 2013 which were failures of the strategy similar to the discussion in Section 4.1.2.

Before 2009, Japanese stock market also suffered from the global financial crisis, so the natural growth of index was below -10% (Figure 14 in Appendices), resulting an exaggerated positive excess return. Again, Bollinger Bands derived from long term analysis equipped the ability to generate profit during adverse environment, due to the training data from the long recession in Japan. Although the strategy could produce positive return in such adverse circumstance, investors might probably be reluctant to investing on the risky downtrend.

However, the stock market did not recover immediately after 2009 like market in the United States. The index fluctuated at the historical low level until 2013 (Figure 13 in Appendices), providing many points of trend reversals for Bollinger Bands to identify and trade profitably. This behaviour is similar to trading on Hang Sang Index at that period, but suggested strategy could not be concluded due to insufficient samples. After 2013, the index grew steadily, and the trend could not be predicted with the previous data, so the strategy did not perform well in that period, akin to the failure observed in Section 4.1.2.

4.2. Suggested Strategies for Trading with Bollinger Bands

The trading simulation on the three selected indices suggested the essentiality of consistent trend movement to the performance of Bollinger Bands. Since this technical analysis tool trades on trend reversals, it strongly relies on a periodic fluctuation within certain range to suggest profitable trading.

While a pair of Bollinger Bands with short time frame (smaller than 60 days) and small SD multiplier (smaller than 1) can be used to exploit investment opportunities in short term fluctuation of a consistent trend (examples suggested by the algorithm are shown by green bands in Figure 9, 11 and 13), it is open to risk of market crashes and recovery which induce new trend behaviours.

To avoid such risk, Bollinger Bands with long time frame (more than 120 days) and large SD multiplier (2.2 ~ 3.2) can be employed instead to trade only at major peaks and tough in market cycles (examples suggested by the algorithm are shown by blue bands in Figure 9, 11 and 13). However, this is a passive investment strategy as the return is merely the natural growth of the financial instruments between the crises, and short term trading opportunities before the potential crisis are forgone.

Trade-off between return and risk is always a dilemma of investment. Further studies on incorporating multiple technical analysis tools with Bollinger Bands can be done to investigate methods of risk minimization in short term and return maximization in long term.

5. Conclusion

Bollinger Bands is a technical analysis tool indicating trend reversal with the measure of relative price level. This study suggested two derivatives of the tool, by calculating the data with exponential weight, and constructing upper bands and lower bands with different parameters. The strategy can provide considerable return above the overall stock market performance in Hong Kong, but more investigation shall be made for the trading in America and Japanese stock markets. The suggested Bollinger Bands to trace short term fluctuation has a time frame shorter than 60 days and a SD multiplier smaller than 1, while the bands for forecasting crises has a time frame over 120 days and a SD multiplier ranged from 2.2 to 3.2.

6. Reference

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7. Appendices

Table 2 Short Term Bollinger Bands with the Highest Excess Return in Trading Tests for HSI

	Upper Band			Lower Band				
Starting Year of Trading Test	Type	Time Frame (days)	SD multiplier	Type	Time Frame (days)	SD multiplier	Number of trades made	Excess Return (%)
2006	Standard	70	0.1	EW	240	1.2	27	9.71
2007	Standard	310	0.1	Standard	310	0.1	23	-0.81
2008	EW	230	0.1	EW	230	0.1	47	3.28
2009	Standard	240	1.0	Standard	30	0.2	69	3.69
2010	EW	20	0.1	EW	90	0.2	105	2.38
2011	EW	100	2.3	EW	100	2.3	3	4.88
2012	Standard	120	0.5	EW	90	1.0	41	1.87
2013	Standard	50	1.5	Standard	270	3.6	40	5.74
2014	EW	40	0.8	Standard	270	3.6	54	5.98
2015	EW	30	0.2	Standard	60	1.0	89	2.71

Table 3 Medium Term Bollinger Bands with the Highest Excess Return in Trading Tests for HSI

	Upper Band			Lower Band				
Starting Year of Trading Test	Type	Time Frame (days)	SD multiplier	Type	Time Frame (days)	SD multiplier	Number of trades made	Excess Return (%)
2006	EW	10	0.1	EW	10	0.1	148	8.00
2007	Standard	300	0.2	Standard	40	0.6	61	7.57
2008	Standard	300	0.2	Standard	350	2.0	31	5.83
2009	Standard	50	0.1	EW	50	0.5	77	4.15
2010	EW	20	0.1	EW	220	0.7	95	2.47
2011	Standard	130	0.3	Standard	30	0.2	66	3.26
2012	EW	240	0.6	EW	240	0.6	59	-1.85
2013	Standard	130	0.3	Standard	30	0.2	68	8.71
2014	EW	20	0.2	Standard	110	3.0	96	8.33
2015	Standard	70	0.9	EW	60	1.2	53	1.04

Table 4 Long Term Bollinger Bands with the Highest Excess Return in Trading Tests for HSI

	Upper Band			Lower Band				
Starting Year of Trading Test	Type	Time Frame (days)	SD multiplier	Type	Time Frame (days)	SD multiplier	Number of trades made	Excess Return (%)
2006	Standard	10	0.1	EW	10	0.4	159	5.45
2007	Standard	10	0.1	EW	10	0.4	171	4.07
2008	Standard	10	0.1	Standard	10	0.1	141	7.49
2009	Standard	10	0.1	Standard	30	0.2	125	6.42
2010	EW	360	2.2	EW	360	2.2	1	8.41
2011	Standard	200	0.9	Standard	200	0.9	55	2.98
2012	Standard	110	0.6	Standard	260	0.9	50	-1.19
2013	EW	20	0.1	Standard	200	0.9	92	3.71
2014	Standard	60	0.9	Standard	60	0.9	62	12.75
2015	Standard	60	0.8	Standard	60	0.8	65	5.63

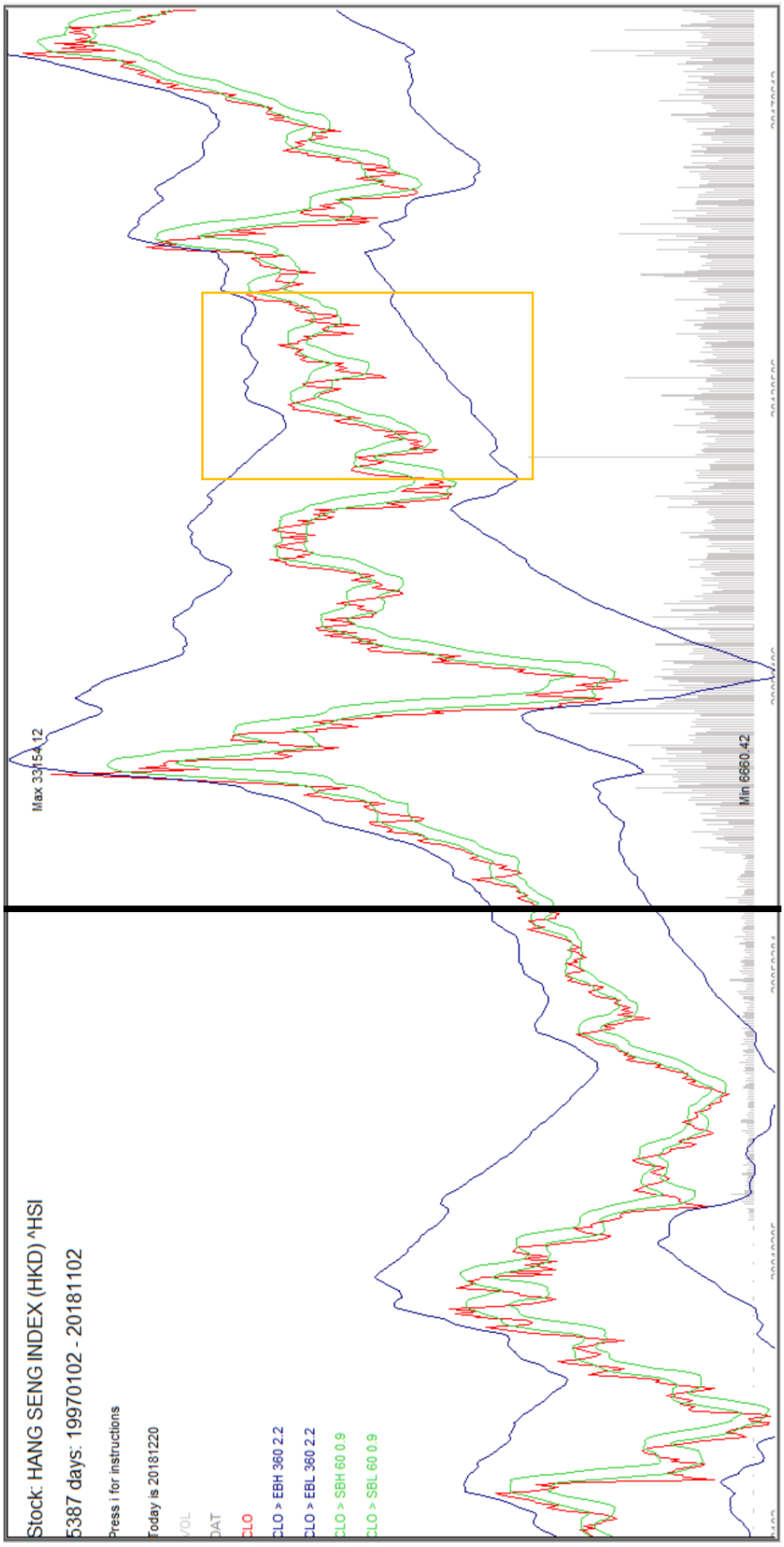

Figure 9 Historical Data of Hang Sang Index Plotted with 2 Suggested Bollinger Bands
Black line indicates the beginning of 2006

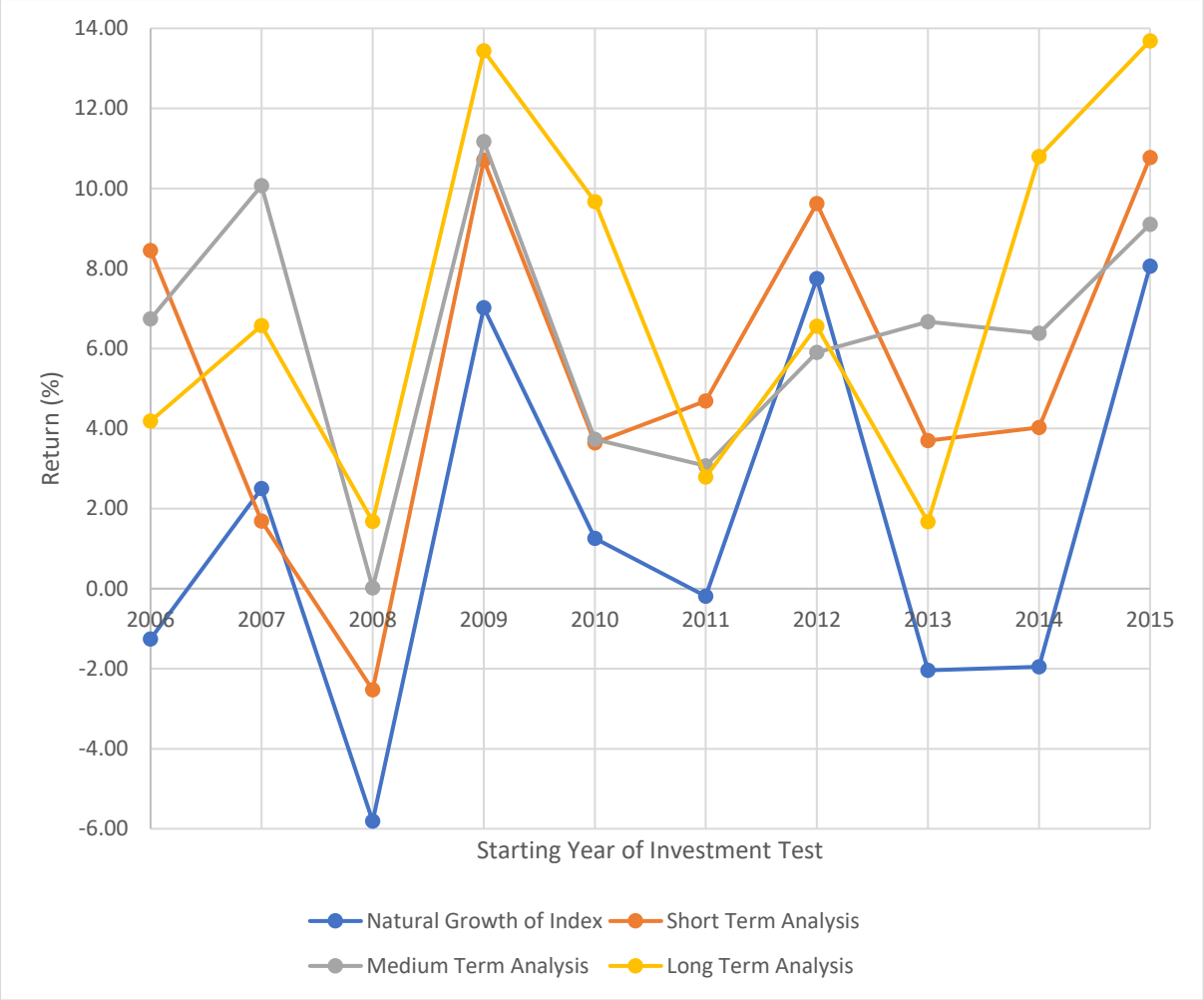

Figure 10 Annual Return of 3-years Investments on Hang Sang Index from 2006 to 2018

Table 5 Short Term Bollinger Bands with the Highest Excess Return in Trading Tests for S&P 500

	Upper Band			Lower Band				
Starting Year of Trading Test	Type	Time Frame (days)	SD multiplier	Type	Time Frame (days)	SD multiplier	Number of trades made	Excess Return (%)
2006	Standard	80	2.6	Standard	80	2.6	17	-12.09
2007	Standard	340	0.1	EW	30	2.1	15	-6.63
2008	EW	360	0.1	Standard	90	3.1	29	-0.15
2009	EW	190	0.6	EW	190	0.6	43	11.27
2010	Standard	130	0.5	EW	50	2.6	43	1.15
2011	EW	50	2.6	EW	50	2.6	3	13.76
2012	Standard	70	2.9	Standard	70	2.9	3	15.85
2013	EW	30	1.5	EW	30	1.5	33	7.59
2014	EW	70	1.9	EW	70	1.9	16	5.49
2015	EW	30	2.0	EW	30	2.0	9	10.74

Table 6 Medium Term Bollinger Bands with the Highest Excess Return in Trading Tests for S&P 500

	Upper Band			Lower Band				
Starting Year of Trading Test	Type	Time Frame (days)	SD multiplier	Type	Time Frame (days)	SD multiplier	Number of trades made	Excess Return (%)
2006	EW	300	0.1	EW	140	2.6	27	-2.85
2007	Standard	270	3.4	Standard	270	3.4	7	-0.62
2008	Standard	340	0.1	Standard	270	3.4	11	1.57
2009	Standard	200	0.9	Standard	200	0.9	41	6.58
2010	Standard	90	0.1	EW	200	0.5	55	5.45
2011	EW	220	0.6	EW	220	0.6	43	9.95
2012	Standard	120	0.7	Standard	80	0.1	47	8.71
2013	Standard	120	0.7	EW	120	3.2	62	4.13
2014	EW	120	3.2	EW	120	3.2	1	4.89
2015	Standard	210	0.2	EW	40	1.7	29	9.49

Table 7 Long Term Bollinger Bands with the Highest Excess Return in Trading Tests for S&P 500

	Upper Band			Lower Band				
Starting Year of Trading Test	Type	Time Frame (days)	SD multiplier	Type	Time Frame (days)	SD multiplier	Number of trades made	Excess Return (%)
2006	EW	200	0.1	Standard	330	0.5	36	-0.65
2007	Standard	340	0.1	EW	310	0.3	13	4.10
2008	Standard	270	3.4	Standard	270	3.4	7	3.45
2009	Standard	340	0.1	Standard	350	0.3	33	-0.51
2010	Standard	280	0.3	Standard	280	0.3	37	3.60
2011	Standard	340	0.1	EW	340	0.5	33	7.58
2012	Standard	210	0.2	Standard	340	1.3	9	14.47
2013	Standard	20	2.7	Standard	20	2.7	15	8.85
2014	EW	120	3.2	EW	120	3.2	1	4.89
2015	EW	120	3.2	EW	120	3.2	1	11.29

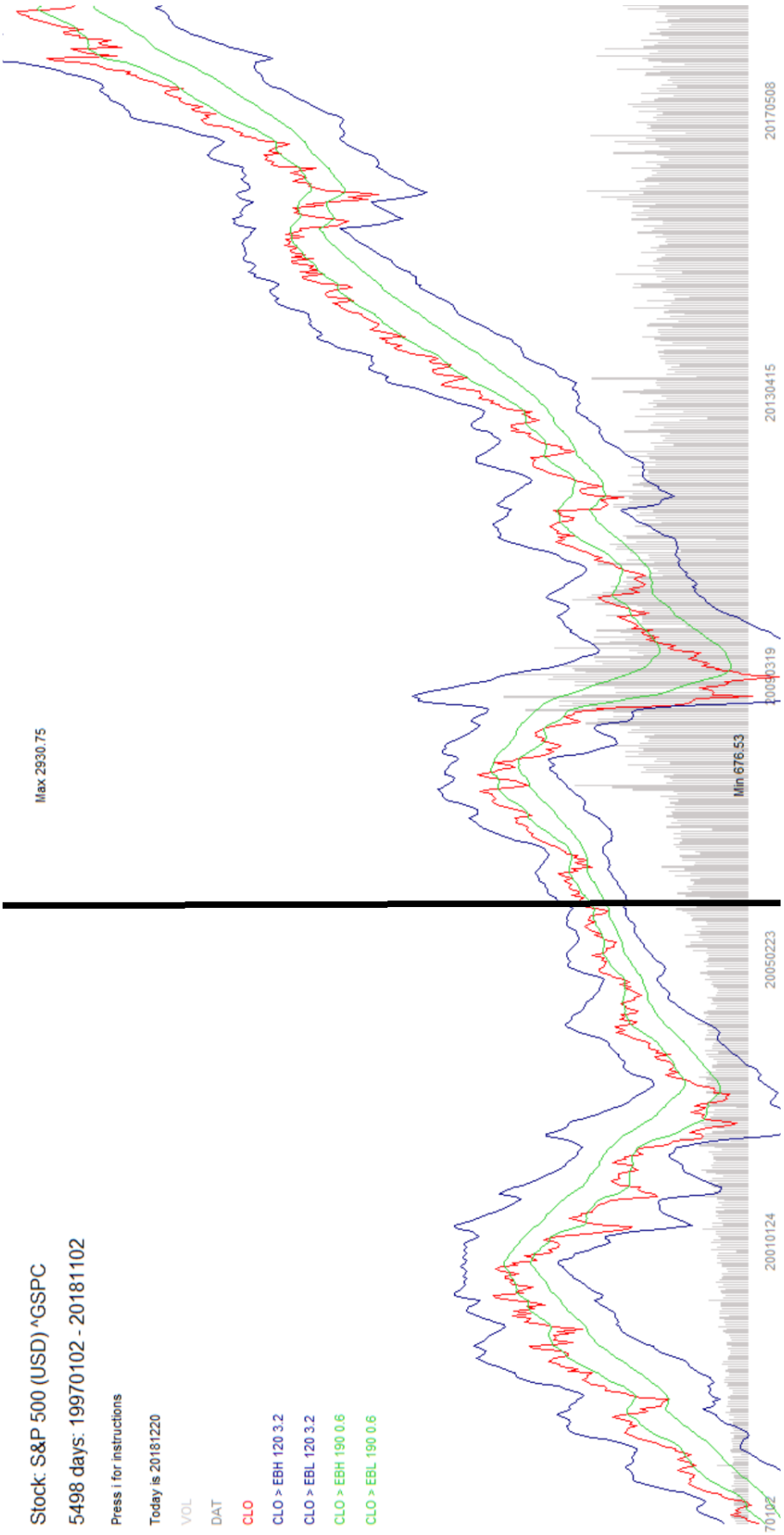

Figure 11 Historical Data of S&P 500 Plotted with 2 Suggested Bollinger Bands
Black line indicates the beginning of 2006

Figure 12 Annual Return of 3-years Investments on S&P500 from 2006 to 2018

Table 8 Short Term Bollinger Bands with the Highest Excess Return in Trading Tests for Nikkei 225

	Upper Band			Lower Band				
Starting Year of Trading Test	Type	Time Frame (days)	SD multiplier	Type	Time Frame (days)	SD multiplier	Number of trades made	Excess Return (%)
2006	Standard	70	0.4	EW	10	1.4	75	-14.84
2007	EW	150	0.8	EW	150	0.8	70	0.90
2008	EW	260	0.2	EW	260	0.2	33	-1.90
2009	EW	280	0.2	EW	270	0.2	34	-0.23
2010	EW	130	0.4	EW	130	0.4	61	4.28
2011	EW	90	0.4	Standard	170	3.3	45	12.18
2012	EW	70	0.3	Standard	10	1.8	81	28.14
2013	Standard	60	1.3	EW	20	1.4	73	18.64
2014	Standard	40	2.7	Standard	40	2.7	15	11.57
2015	EW	50	0.8	EW	50	0.8	110	5.29

Table 9 Medium Term Bollinger Bands with the Highest Excess Return in Trading Tests for Nikkei 225

	Upper Band			Lower Band				
Starting Year of Trading Test	Type	Time Frame (days)	SD multiplier	Type	Time Frame (days)	SD multiplier	Number of trades made	Excess Return (%)
2006	EW	80	0.3	Standard	60	3.3	49	-1.33
2007	EW	80	0.3	Standard	60	3.3	48	-3.35
2008	Standard	80	0.4	EW	50	2.1	35	-0.91
2009	EW	80	0.3	Standard	180	1.5	61	-0.19
2010	EW	130	0.4	EW	130	0.4	61	4.28
2011	EW	40	0.3	Standard	110	0.5	55	20.36
2012	Standard	320	0.3	Standard	320	0.3	39	15.56
2013	EW	40	0.3	Standard	80	0.2	83	9.91
2014	Standard	360	1.1	Standard	360	1.1	32	5.59
2015	Standard	20	0.5	EW	10	1.5	100	5.43

Table 10 Long Term Bollinger Bands with the Highest Excess Return in Trading Tests for Nikkei 225

	Upper Band			Lower Band				
Starting Year of Trading Test	Type	Time Frame (days)	SD multiplier	Type	Time Frame (days)	SD multiplier	Number of trades made	Excess Return (%)
2006	Standard	160	0.1	Standard	50	3.6	24	-0.11
2007	Standard	310	0.1	EW	30	2.3	21	3.05
2008	Standard	170	0.3	EW	30	2.3	29	-1.93
2009	Standard	130	0.9	EW	280	0.2	38	4.32
2010	Standard	80	0.4	Standard	140	0.1	57	2.91
2011	EW	70	0.3	EW	120	0.4	55	16.83
2012	EW	70	0.3	Standard	30	1.5	67	20.31
2013	EW	40	0.3	Standard	30	1.5	93	6.41
2014	EW	220	0.4	EW	220	0.4	55	3.71
2015	EW	10	1.9	EW	10	1.9	3	9.45

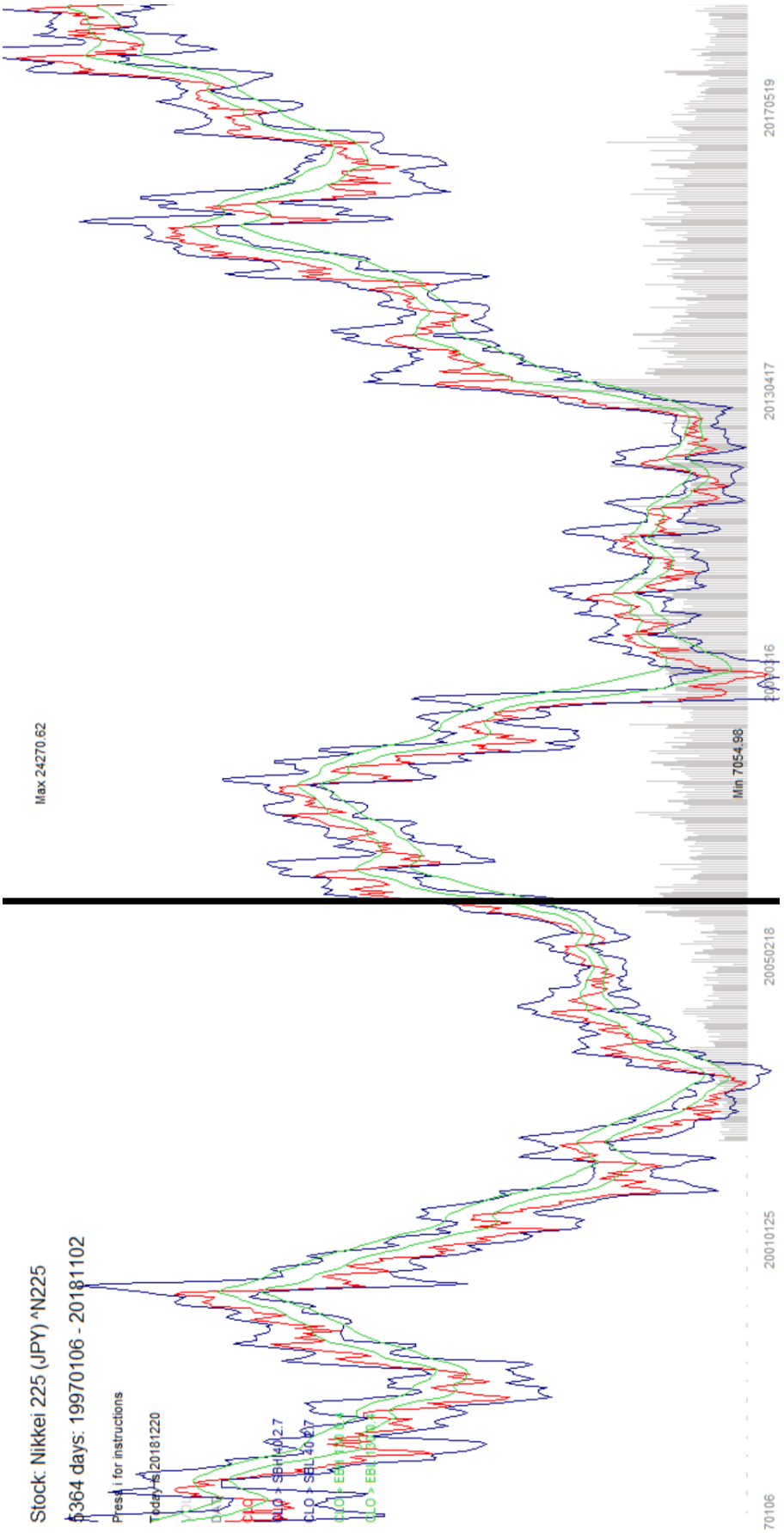

Figure 13 Historical Data of Nikkei 225 Plotted with 2 Suggested Bollinger Bands
Black line indicates the beginning of 2006

Figure 14 Annual Return of 3-years Investments on Nikkei 225 from 2006 to 2018