A Noncooperative Spectrum Sensing Game with Maximum Network Throughput

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***Abstract*—In this paper, we consider a noncooperative cognitive radio network with M selfish secondary users (SUs) opportunistically access N licensed channels. Every SU chooses one channel to sense and subsequently compete to access (based on the sensing outcome) to obtain the channel utility. Different channels may have different utilities. Each SU selfishly makes a sensing decision to maximize its obtained utility. The objective is to design an optimal sensing policy with maximum network throughput. This problem is formulated as a noncooperative game where a stable sensing policy reaches a Nash Equilibrium (NE). A novel greedy algorithm with great efficiency is proposed to calculate all pure-strategy NE for a large class of utility functions. By slight modification, the algorithm is able to reach an optimal pure-strategy NE with the maximum network throughput. The algorithm can be practically implemented as a MAC protocol in a distributed way with negligible communication overhead.**

I. Introduction

Cognitive radio networks (CRNs) have recently emerged as a revolutionary technique to mitigate the underutilization of wireless spectrum [1]. Studies show that with conventional *static* spectrum allocation policy, even in the most crowded area, 62% of spectrum remain unutilized [2]. To address this problem, in CRN, *secondary users* (SUs) are allowed to opportunistically access the licensed spectrum without introducing disruptive interferences to the *primary users* (PUs) [3].

One key enabling technique in realizing CRN is *spectrum sensing policy*. Usually, it is too expensive for SUs to scan all channels to discover all access opportunities. Partially sensing a small portion of the channels to access is thus a necessity. It is the spectrum sensing policy that decides which portion should be selected since they are more likely sensed to be idle than others. The research community has proposed several sensing policies to maximize the expected network throughput: [4]–[7] discuss the scenario where there is only a single SU in CRN, while [8]–[11] discuss the scenario of multiple SUs under the assumption that all SUs cooperate with each other in an unselfish way. However, in CRN, different SUs may come from different network groups, and thus may not be willing to be unselfish: one spectrum preferred by one SU may also be desired by other SUs. Although there are several game theoretic works to address the selfish issue in the context of CRN, most of them focus on analyzing power control or spectrum pricing/auction [12]–[14]. To the best of our knowledge, only a few literatures have studied selfish sensing policy under a simple scenario: [15] studies two-user two-channel setting. However, the proposed threshold-based sensing policy cannot achieve a Nash Equilibrium.

In this paper, we are motivated to consider sensing policies for M selfish SUs that are allowed to opportunistically access N licensed channels. Every SU adopts a "listen before talk" policy, i.e., chooses one channel to sense (due to limited sensing/accessing capability) and subsequently comepte to access (based on the sensing outcome). If multiple SUs select the same channel, with the same probability, only one of them accesses and obtains the channel utility. Every SU selfishly makes a sensing decision to maximize its obtained utility. The objective is to design an optimal sensing policy with maximum network throughput. This spectrum sensing problem can be formulated as a *noncooperative game*, where a stable sensing policy reaches a *Nash Equilibrium* (NE). Effectively computing NE is, in general, indeed hard [16]. A typical workaround is to guess a starting point and then iteratively adjust it to converge to a NE [12]. However, practically using this iterative algorithm to find a sensing policy is questionable in that 1) the iteration may converge slowly so that sensing/access has to be postponed for a long time; 2) the iteration may not converge to an optimal NE with maximum network throughput; 3) the communication overhead may become overwhelming since a large amount of information has to be exchanged during iteration provided that no central controller exists.

In this study, we present an efficient *greedy algorithm* to find an optimal selfish sensing policy with maximum network throughput. Without any iterations, the proposed algorithm makes a sensing decision for one SU in each step, and terminates at the Mth step, reaching an optimal *pure-strategy* NE with maximum network throughput. The algorithm is efficient in that the time and space complexity are $\mathcal{O}(M \log N)$ and $\mathcal{O}(N)$, respectively. The main contributions of this research are as follows:

- For a noncooperative spectrum sensing game, a greedy algorithm is proposed with simple structure to effectively find a pure-strategy NE. It is shown that, interestingly, a sensing policy is a pure-strategy NE *if and only if* it can be calculated by the algorithm.
- It is proved that by slight modification, the algorithm can lead all SUs to an optimal pure-strategy NE with the maximum network throughput.
- The utility function is defined in a *general* form so that the algorithm and analysis are effective for a large class

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of utility functions.

- A practical MAC protocol is given to show that the algorithm can be practically applied in a distributed way with negligible communication overhead.

The rest of this paper is organized as follows. After the system model is presented in Section II, the problem is formulated as a noncooperative game in Section III. In Section IV, the algorithm and the related theorems are formally stated. In Section V, a practical MAC protocol is given to implement the algorithm in a distributed way. Section VI concludes the whole paper.

II. System Model

Consider a CRN with M selfish SUs that are allowed to opportunistically access N channels licensed to PUs. All these M SUs are within each other's interference range. Let $\Omega_u \triangleq \{1, 2, \ldots, M\}$ and $\Omega_c \triangleq \{1, 2, \ldots, N\}$ be the set of SUs and channels, respectively. Denote $\mathbf{D} \triangleq [D_1, D_2, \ldots, D_N]^T$ as the *delivery rate vector*, where D_i is the delivery rate of channel i. Note that D_i is determined by the activities of PUs in channel i and can be defined as any specific form of delivery rate. For example: D_i can be either $D_i \triangleq B_i P_i$ or $D_i \triangleq P_i B_i \log(1 + \text{SNR}_i)$ where B_i and P_i are the bandwidth and available probability of channel i, respectively. Based on the observation of channel utilizations, $\mathbf{D}$ is a common knowledge to all SUs.

To ensure that the interference to PUs is within a tolerable level, every SU adopts a "listen before talk" (LBT) policy. That is, when an SU wants to transmit, it first decides which channel should be selected to sense. Here we assume that one SU can sense only *one* channel at a time due to limited sensing/accessing capability [7]. The SU then starts sensing on the selected channel. If the channel is sensed to be idle, the SU transmits using carrier sensing. Note that if multiple SUs decide to access the same channel, say, channel k, a *fair* competition (i.e., every contender wins with the same probability) begins and only *one* of them wins to transmit on the channel with the corresponding delivery rate, say, D_k.

III. Problem Formulation

In a "noncooperative" CRN, each SU behaves selfishly in preferring maximizing its own interest. This problem can be formulated as a *noncooperative game* $\mathcal{G}(\Omega_u, \{\mathbf{S}_i\}, \{u_i\})$. The players (i.e., Ω_u) are M SUs; the strategies (i.e., $\{\mathbf{S}_i\}$) are defined as the channels selected by SUs to sense[1]; and the utility functions (i.e., $\{u_i\}$) depend on the strategy profile. We will use the term "player" interchangeably with "SU" in the following context.

A. Strategy Profile

For mathematical convenience, an SU is allowed to select no channel to sense. Let $\Omega_e \triangleq \{\mathbf{0}, \mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_N\}$, where $\mathbf{0} \triangleq [0, \ldots, 0]_N^T$ and $\mathbf{e}_i$ is a N-dimensional unit vector with the ith component being "1".

Definition 1: $\mathbf{S}_i$ is a *pure strategy* of player i ($\forall i \in \Omega_u$) if $\mathbf{S}_i \in \Omega_e$. Player i is said to select channel k if $\mathbf{S}_i = \mathbf{e}_k, k \in \Omega_c$, or nothing if $\mathbf{S}_i = \mathbf{0}$.

Definition 2: A *pure-strategy profile* of game $\mathcal{G}$ is a $N \times M$ matrix, $\mathbf{S} \triangleq [\mathbf{S}_1, \mathbf{S}_2, \ldots, \mathbf{S}_M]$, where $\mathbf{S}_i$ is a pure strategy of player i ($\forall i \in \Omega_u$).

Usually, $\mathbf{S}$ can also be denoted by $(\mathbf{S}_i, \mathbf{S}_{-i})$, where $\mathbf{S}_{-i}$ is the set of all strategy profiles of players other than i, i.e., $\mathbf{S}_{-i} \triangleq \{\mathbf{S}_1, \ldots, \mathbf{S}_{i-1}, \mathbf{S}_{i+1}, \ldots, \mathbf{S}_M\}$.

B. Utility Function

Utility function is defined as the utility that a player can receive under certain strategy profile [17]. In $\mathcal{G}$, let $u_i(\mathbf{S}_i, \mathbf{S}_{-i})$, also denoted by $u_i(\mathbf{S})$, be the untility funcion of player i under a pure-strategy profile $(\mathbf{S}_i, \mathbf{S}_{-i})$, also denoted by $\mathbf{S}$.

One important consideration of this work is to define the utility function in a *general* form. Given a pure-strategy profile $\mathbf{S}$, if player i selects nothing, say, $\mathbf{S}_i = \mathbf{0}$, define $u_i(\mathbf{S}_i, \mathbf{S}_{-i}) \triangleq -\infty$. Otherwise, assume $\mathbf{S}_i = \mathbf{e}_k$, then $u_i(\mathbf{S}_i, \mathbf{S}_{-i})$ depends on the following two things: 1) D_k, the delivery rate of channel k, and 2) $\mathbf{S}_{-i}$, other players' strategies. Assume there are m_k SUs that decide to sense channel k under $\mathbf{S}$. Reasonably, due to collisions, the more m_k is, the less $u_i(\mathbf{S}_i, \mathbf{S}_{-i})$ will be. Thereby, it can be denoted that $u_i(\mathbf{S}_i, \mathbf{S}_{-i}) \triangleq U_i(k, m_k, D_k)$, where $U_i(k, m_k, D_k)$ is strictly decreasing with respect to m_k. Furthermore, for all these m_k SUs, since the access competition is *fair*, everyone obtains the same utility. Also notice that D_k is a constant, $U_i(k, m_k, D_k)$ can then be simplified to $U(k, m_k)$, where $U(k, m_k)$ is strictly decreasing with respect to m_k. We also assume that $U(k, m_k)$ is known a priori to each SU.

Before we formally define $u_i(\mathbf{S}_i, \mathbf{S}_{-i})$, we introduce the following assistant vectors. Let $\mathbf{I} \triangleq [1, 2, \ldots, N]^T$, $\mathbf{J} \triangleq [1, \ldots, 1]_M^T$, and $\mathbf{c}_i$ be an M-dimensional unit vector with the ith component being "1". Then player i selects channel k to sense[2], where $k = \mathbf{I}^T \cdot \mathbf{S} \cdot \mathbf{c}_i$. And m_k players make the same sensing decision of k, where $m_k = \mathbf{e}_k^T \cdot \mathbf{S} \cdot \mathbf{J}$ $(k > 0)$.

Definition 3: Given a pure-strategy profile $\mathbf{S}$, for all $i \in \Omega_u$, define the *utility function* of player i under $\mathbf{S}$ by

$$u_i(\mathbf{S}_i, \mathbf{S}_{-i}) \triangleq \begin{cases} U(k, m_k), & \text{if } k > 0, \\ -\infty, & \text{if } k = 0, \end{cases} \tag{1}$$

where $U(k, m_k)$ is a function that is strictly decreasing with respect to m_k, i.e., $U(k, m_k) > U(k, m_k') \Leftrightarrow m_k < m_k'$.

As a simple example, $U(k, m_k)$ can be $\frac{1}{m_k} D_k$.

C. Game Formulation

So far we have formulated a noncooperative game $\mathcal{G}(\Omega_u, \{\mathbf{S}_i\}, \{u_i\})$. Since each player is selfish, a *stable* sensing policy must reach a NE. That is, no player can improve its utility by unilaterally changing its own strategy. The formal definition of NE is given as follows [17].

[1] In this paper, we limit our discussion to pure strategies only.

[2] $k = 0$ indicates that player i selects nothing to sense

Definition 4: In $\mathcal{G}$, a pure-strategy profile $\mathbf{S}^*$ is a *pure-strategy NE* if, for each player i ($i \in \Omega_u$),

$$u_i(\mathbf{S}_i^*, \mathbf{S}_{-i}^*) \geq u_i(\mathbf{S}_i, \mathbf{S}_{-i}^*), \quad \forall \mathbf{S}_i \in \Omega_e. \tag{2}$$

If there exists several pure-strategy NE in $\mathcal{G}$, it is desirable to reach the one with the maximum achievable network throughput, i.e., the sum-rate of all transmissions in SUs. For channel i, if no SU decides to sense/access it, its delivery rate is zero; if there is at least one SU that decides to sense/access it, after the competition, only one can finally access it with delivery rate D_i. Therefore, the network throughput (sum-rate) can be solely determined by the strategy profile.

Definition 5: Given a pure-strategy profile $\mathbf{S}$, the *network throughput* is defined as

$$R(\mathbf{S}) \triangleq \sum_{i=1}^{N} I_i(\mathbf{S}) D_i \quad , \tag{3}$$

where for all $i = 1, 2, \ldots, N$,

$$I_i(\mathbf{S}) \triangleq \begin{cases} 1, & \text{if } \exists j \in \Omega_u : \mathbf{S}_j = \mathbf{e}_i, \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

The objective is to reach an optimal pure-strategy NE with the maximum network throughput:

$$\begin{aligned} \max_{\mathbf{S}^*} \quad & R(\mathbf{S}^*) \\ \text{s.t.} \quad & \mathbf{S}^* \text{ is a pure-strategy NE in } \mathcal{G}. \end{aligned} \tag{5}$$

Before we proceed to the solution of (5) in Section IV, Theorem 1 guarantees the existence of pure-strategy NE in $\mathcal{G}$. Its proof will be clear after introducing Theorem 2.

Theorem 1: $\mathcal{G}$ has at least one pure-strategy NE.

IV. Greedy Algorithm

This section formally states the proposed *greedy algorithm* to solve (5) effectively. First, a *basic greedy algorithm* (BGA) that is sufficient to calculate *every* pure-strategy NE is given. Then, based on BGA, an *optimal greedy algorithm* (OGA) is proposed to solve (5) with great efficiency.

A. Calculate Every Pure-Strategy NE by BGA

Let $\phi : \Omega_u \mapsto \Omega_u$ be an arbitrary one-on-one mapping. BGA is proposed as follows to calculate every pure-strategy NE:

Algorithm 1 Basic Greedy Algorithm

Input: $U(k, m_k)$ and $\phi : \Omega_u \mapsto \Omega_u$

Output: $\mathbf{S}^*$, a pure-strategy NE in $\mathcal{G}(\Omega_u, \{\mathbf{S}_i\}, \{u_i\})$

1: $m_i^0 \leftarrow 0, \forall i \in \Omega_c$
2: **for** $i = 1$ to M **do**
3: $\quad k \leftarrow \arg\max_{j \in \Omega_c} U(j, m_j^{i-1} + 1)$
4: $\quad m_k^i \leftarrow m_k^{i-1} + 1$
5: $\quad m_j^i \leftarrow m_j^{i-1}, \forall j \in \Omega_c \setminus \{k\}$
6: $\quad \mathbf{S}_{\phi(i)}^* \leftarrow \mathbf{e}_k$
7: **end for**
8: **return** $\mathbf{S}^*$

Remark 1: After the ith loop, there are m_k^i SUs selecting channel k. Specifically, at last every SU selecting channel k shares the same utility function, i.e., $U(k, m_k^M)$.

Remark 2: In line 3, if there are multiple k's achieving the maximum value, everyone is qualified to be selected.

The main idea of BGA is to let every player sequentially make a decision to maximize the immediate utility, without concerning the impact to the future. Though the idea is simple, BGA is sufficient to calculate every pure-strategy NE in $\mathcal{G}$ by inputting different ϕ's and selecting different k's in line 3 (if multiple k's are qualified for selection). Before the formal proof is given, let's take a look at a toy example of a 3-player 3-channel game as follows:

Example 1: For a 3-player 3-channel game $\mathcal{G}$, assume $D_1 = 10, D_2 = 8, D_3 = 16$, and $U(k, m_k) = \frac{1}{m_k} D_k, k = 1, 2, 3$. Let $\phi(1) = 1, \phi(2) = 2$ and $\phi(3) = 3$. It can be found that after the first loop, $k = 3, m_1^1 = 0, m_2^1 = 0, m_3^1 = 1$ and $\mathbf{S}_1^* = \mathbf{e}_3$; after the second loop, $k = 1, m_1^2 = 1, m_2^2 = 0, m_3^2 = 1$ and $\mathbf{S}_2^* = \mathbf{e}_1$; in the final loop, both $k = 2$ and $k = 3$ are qualified for selection. Clearly, the two calculated pure-strategy profiles $\mathbf{S}^* = [\mathbf{e}_3, \mathbf{e}_1, \mathbf{e}_2]$ and $\mathbf{S}^{**} = [\mathbf{e}_3, \mathbf{e}_1, \mathbf{e}_3]$ are both pure-strategy NE. Similarly, changing ϕ to different mappings results in different pure-strategy NE. These calculation results are all pure-strategy NE in $\mathcal{G}$.

B. Necessity and Sufficiency

One interesting result we get is that: a strategy profile is a pure-strategy NE *if and only if* it can be calculated by BGA. This section formally proves the statement by proposing Theorem 2 and Theorem 3. We omit the proof of all lemmas since they are straightforward.

Lemma 1: In BGA, $m_k^0 \leq m_k^1 \leq \cdots \leq m_k^M, \forall k \in \Omega_c$.

Lemma 2: In BGA, the following three conditions are equivalent to each other, $\forall i \in \Omega_u, \forall k \in \Omega_c$.

1) $m_k^{i-1} < m_k^i$,
2) $m_k^i = m_k^{i-1} + 1$,
3) $\mathbf{S}_{\phi(i)}^* = \mathbf{e}_k$,

and they all imply that:

$$U(k, m_k^{i-1} + 1) = \max_{j \in \Omega_c} U(j, m_j^{i-1} + 1). \tag{6}$$

Theorem 2 (Sufficiency): A strategy profile $\mathbf{S}^*$ calculated by BGA is a pure-strategy NE in $\mathcal{G}(\Omega_u, \{\mathbf{S}_i\}, \{u_i\})$.

Proof: Without loss of generality, it is sufficient to consider $\phi(i) = i, \forall i \in \Omega_u$. For an $i \in \Omega_u$, assume $\mathbf{S}_i^* = \mathbf{e}_k, k \in \Omega_c$. We assert: $u_i(\mathbf{S}_i^*, \mathbf{S}_{-i}^*) \geq u_i(\mathbf{S}_i, \mathbf{S}_{-i}^*), \forall \mathbf{S}_i \in \Omega_e$. Clearly, the assertion is guaranteed if $\mathbf{S}_i = \mathbf{0}$. If $\mathbf{S}_i = \mathbf{e}_l$, then there will be $m_l^M + 1$ players selecting channel l under $(\mathbf{S}_i, \mathbf{S}_{-i}^*)$. Hence, $u_i(\mathbf{S}_i, \mathbf{S}_{-i}^*) = U(l, m_l^M + 1)$. Since $m_k^i \leq m_k^M$ (Lemma 1), it is sufficient to consider the following two cases:

Case 1, $m_k^i = m_k^M$: Since $\mathbf{S}_i^* = \mathbf{e}_k$, according to Lemma 2, we know that $m_k^i = m_k^{i-1} + 1$. Therefore,

$$\begin{aligned} u_i(\mathbf{S}_i^*, \mathbf{S}_{-i}^*) = U(k, m_k^M) &= U(k, m_k^{i-1} + 1) \\ &\geq U(l, m_l^{i-1} + 1) \\ &\geq U(l, m_l^M + 1) = u_i(\mathbf{S}_i, \mathbf{S}_{-i}^*), \end{aligned} \tag{7}$$

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where the first and the second "$\geq$" derive from Lemma 2 and Lemma 1, respectively.

Case 2, $m_k^i < m_k^M$: Let $j = \min\{s \mid m_k^s = m_k^M, s \in \Omega_u\}$. According to Lemma 1 and Lemma 2, we have $m_k^M = m_k^j = m_k^{j-1} + 1$. Then,

$$\begin{aligned} u_i(\mathbf{S}_i^*, \mathbf{S}_{-i}^*) &= U(k, m_k^M) = U(k, m_k^{j-1} + 1) \\ &\geq U(l, m_l^{j-1} + 1) \\ &\geq U(l, m_l^M + 1) = u_i(\mathbf{S}_i, \mathbf{S}_{-i}^*). \end{aligned} \tag{8}$$

Therefore, the assertion is guaranteed. Also notice that $\mathbf{S}^*$ is a pure-strategy profile, we hence conclude the proof. ■

To show that any NE in $\mathcal{G}$ is a calculation result of BGA, we propose Algorithm 2 served as an assistant step in the proof[3].

Algorithm 2 Assistant Algorithm in Proving Theorem 3

Input: Utility functions and $\mathbf{S}$, a pure-strategy profile in $\mathcal{G}$
Output: A one-on-one mapping $\phi : \Omega_u \mapsto \Omega_u$

1: $\Omega_r^0 \leftarrow \Omega_u$
2: $\mathbf{S}^0 \leftarrow \mathbf{0}_{N\times M}$
3: $\mathbf{A}_i \leftarrow \mathbf{S}_i \cdot \mathbf{c}_i^T, \forall i \in \Omega_u$
4: **for** $i = 1$ to M **do**
5: $\quad k \leftarrow \arg\max_{j\in\Omega_r^{i-1}} u_j(\mathbf{S}^{i-1} + \mathbf{A}_j)$
6: $\quad \Omega_r^i \leftarrow \Omega_r^{i-1} \setminus \{k\}$
7: $\quad \mathbf{S}^i \leftarrow \mathbf{S}^{i-1} + \mathbf{A}_k$
8: $\quad \phi(i) \leftarrow k$
9: **end for**
10: **return** ϕ

Lemma 3: In Algorithm 2, for all $i \in \Omega_u$, we have $u_{\phi(i)}(\mathbf{S}^i) \geq u_{\phi(i)}(\mathbf{S}^{i+1}) \geq \ldots \geq u_{\phi(i)}(\mathbf{S}^M) = u_{\phi(i)}(\mathbf{S})$.

Theorem 3 (Necessity): In $\mathcal{G}(\Omega_u, \{\mathbf{S}_i\}, \{u_i\})$, any pure-strategy NE can be calculated by BGA.

Proof: Let $\mathbf{S}^*$ be an arbitrary pure-strategy NE in $\mathcal{G}$. By running Algorithm 2 with input $\mathbf{S}^*$, we can get a one-on-one mapping $\phi : \Omega_u \mapsto \Omega_u$. Then run BGA with input ϕ. We will prove that $\mathbf{S}_{\phi(i)}^*$ can be calculated in the ith loop ($1 \leq i \leq M$) of BGA. Call this statement $P(M)$. We prove this statement by mathematical induction. And the notations $u_i(\mathbf{S})$ and $u_i(\mathbf{S}_i, \mathbf{S}_{-i})$ will be used interchangeably below.

Basis: Show that $P(1)$ holds. Assume $\mathbf{S}_{\phi(1)}^* = \mathbf{e}_k$. Assertion 1: $U(k, m_k^0 + 1) = \max_{j\in\Omega_c} U(j, m_j^0 + 1)$. Otherwise, $\exists l \in \Omega_c : U(k, m_k^0 + 1) < U(l, m_l^0 + 1)$. Now if $\exists t > 1 : \mathbf{S}_{\phi(t)}^* = \mathbf{e}_l$, we have

$$\begin{aligned} u_{\phi(t)}(\mathbf{S}^0 + \mathbf{A}_{\phi(t)}) &= U(l, m_l^0 + 1) \\ &> U(k, m_k^0 + 1) \\ &= u_{\phi(1)}(\mathbf{S}^0 + \mathbf{A}_{\phi(1)}), \end{aligned} \tag{9}$$

which contradicts with line 5 in Algorithm 2. Therefore, no player chooses channel l under $\mathbf{S}^*$. Now if player $\phi(1)$ unilaterally changes its strategy to $\mathbf{S}_{\phi(1)} = \mathbf{e}_l$, we have

$$\begin{aligned} u_{\phi(1)}(\mathbf{S}_{\phi(1)}, \mathbf{S}_{-\phi(1)}^*) &= U(l, 1) = U(l, m_l^0 + 1) \\ &> U(k, m_k^0 + 1) = u_{\phi(1)}(\mathbf{S}^1) \\ &\geq u_{\phi(1)}(\mathbf{S}^M) = u_{\phi(1)}(\mathbf{S}^*) \\ &= u_{\phi(1)}(\mathbf{S}_{\phi(1)}^*, \mathbf{S}_{-\phi(1)}^*), \end{aligned} \tag{10}$$

where "$\geq$" derives from Lemma 3. However, (10) contradicts with the fact that $\mathbf{S}^*$ is a NE. Thus, Assertion 1 is guaranteed, which means that k is qualified to be selected in line 3 in BGA. Thereby, $P(1)$ holds.

Inductive step: Show that if $P(n-1)$ holds, then also $P(n)$ holds ($1 < n \leq M$). Assume $P(n-1)$ holds, then $\mathbf{S}_{\phi(i)}^*$ can be calculated at the ith loop of BGA, $1 \leq i \leq n-1$. Therefore, after $(n-1)$ loops, the strategy profile is $\mathbf{S}^{n-1}$ with m_l^{n-1} players selecting channel l ($l \in \Omega_c$). Assume $\mathbf{S}_{\phi(n)}^* = \mathbf{e}_k$. Assertion 2: $U(k, m_k^{n-1} + 1) = \max_{j\in\Omega_c} U(j, m_j^{n-1} + 1)$. Otherwise, $\exists l \in \Omega_c : U(k, m_k^{n-1} + 1) < U(l, m_l^{n-1} + 1)$. Now if $\exists t > n : \mathbf{S}_{\phi(t)}^* = \mathbf{e}_l$, we have

$$\begin{aligned} u_{\phi(t)}(\mathbf{S}^{n-1} + \mathbf{A}_{\phi(t)}) &= U(l, m_l^{n-1} + 1) \\ &> U(k, m_k^{n-1} + 1) \\ &= u_{\phi(n)}(\mathbf{S}^{n-1} + \mathbf{A}_{\phi(n)}), \end{aligned} \tag{11}$$

which contradicts with line 5 in Algorithm 2. Thereby, there are m_l^{n-1} players selecting channel l under $\mathbf{S}^*$. Now if player $\phi(n)$ unilaterally changes its strategy to $\mathbf{S}_{\phi(n)} = \mathbf{e}_l$, we have

$$\begin{aligned} u_{\phi(n)}(\mathbf{S}_{\phi(n)}, \mathbf{S}_{-\phi(n)}^*) &= U(l, m_l^{n-1} + 1) \\ &> U(k, m_k^{n-1} + 1) = u_{\phi(n)}(\mathbf{S}^n) \\ &\geq u_{\phi(n)}(\mathbf{S}^M) = u_{\phi(n)}(\mathbf{S}^*) \\ &= u_{\phi(n)}(\mathbf{S}_{\phi(n)}^*, \mathbf{S}_{-\phi(n)}^*), \end{aligned} \tag{12}$$

which contradicts with the fact that $\mathbf{S}^*$ is a NE. Thus, Assertion 2 holds, thereby showing that indeed $P(n)$ holds.

Since both the basis and the inductive step are guaranteed, by mathematical induction, we see that $P(M)$ holds. ■

Combining Theorem 2 and Theorem 3, finally we have:

Theorem 4 (Necessity and Sufficiency): In $\mathcal{G}$, $\mathbf{S}^*$ is a pure-strategy NE *if and only if* it can be calculated by BGA.

C. Reach an Optimal Pure-Strategy NE by OGA

So far all pure-strategy NE can be calculated by BGA. However, not everyone is a solution to (5). To see this, take a look at the following 3-player 3-channel game:

Example 2: Consider a 3-player 3-channel game $\mathcal{G}$, where $D_1 = 18, D_2 = 9, D_3 = 9$ and $U(k, m_k) = \frac{1}{m_k} D_k, k = 1, 2, 3$. Let $\phi(1) = 1, \phi(2) = 2, \phi(3) = 3$ and $U(k, m_k)$ be the input of BGA. After the first loop, $\mathbf{S}_1^* = \mathbf{e}_1$. In the second loop, $\mathbf{S}_2^*$ has three different choices, i.e., $\mathbf{e}_1, \mathbf{e}_2$ and $\mathbf{e}_3$. Since anyone is eligible, assume $\mathbf{S}_2^* = \mathbf{e}_2$. Now in the third loop, both $\mathbf{e}_1$ and $\mathbf{e}_3$ are qualified candidates of $\mathbf{S}_3^*$. However, they result in different network throughputs, i.e., $R([\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_1]) = 27$ and $R([\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3]) = 36$.

Naturally, an intuitive idea to improve the reached NE is that: when multiple choices are available in one loop, instead

[3] In line 3, $\mathbf{c}_i$ is defined in Section III-B, and line 3 results in $\mathbf{A}_1 = [\mathbf{S}_1, \ldots, 0]$, $\mathbf{A}_2 = [0, \mathbf{S}_2, \ldots, 0]$, …, and $\mathbf{A}_M = [0, \ldots, \mathbf{S}_M]$.

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of random choices, select the one that can maximize the immediate increase of network throughput. Back to Example 2, in the first loop, $\mathbf{e}_1$, the only choice for $\mathbf{S}_1^*$, brings the immediate throughput increase of 18. In the second loop, the immediate increase brought by $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ are $0, 9, 9$, respectively. Thus either $\mathbf{e}_2$ or $\mathbf{e}_3$ is preferred. Assume $\mathbf{e}_3$ is selected. In the last loop, both $\mathbf{e}_1$ and $\mathbf{e}_2$ are eligible with throughput increase of 0 and 9, respectively. Hence, $\mathbf{S}^* = [\mathbf{e}_1, \mathbf{e}_3, \mathbf{e}_2]$. It is easy to verify that $\mathbf{S}^*$ reaches the maximum network throughput of 36.

Interestingly, although the idea is intuitive, it does provide a solution to (5). Based on this, OGA (*optimal greedy algorithm*), a modified version of BGA, is proposed in the following to reach an optimal NE with maximum network throughput, as guaranteed by Theorem 5:

Algorithm 3 Optimal Greedy Algorithm in Solving (5)

Input: $U(k, m_k)$ and $\phi : \Omega_u \mapsto \Omega_u$
Output: $\widehat{\mathbf{S}}^*$, a solution to (5)

1: $\widehat{\mathbf{S}}^0 \leftarrow \mathbf{0}_{N \times M}$
2: $\widehat{m}_i^0 \leftarrow 0, \forall i \in \Omega_c$
3: **for** $i = 1$ to M **do**
4: $\quad \Omega^i \leftarrow \{p \mid p = \arg\max_{j \in \Omega_c} U(j, \widehat{m}_j^{i-1} + 1)\}$
5: $\quad k \leftarrow \arg\max_{j \in \Omega^i} R(\widehat{\mathbf{S}}^{i-1} + \mathbf{e}_j \cdot \mathbf{c}_{\phi(i)}^T)$
6: $\quad \widehat{\mathbf{S}}^i \leftarrow \widehat{\mathbf{S}}^{i-1} + \mathbf{e}_k \cdot \mathbf{c}_{\phi(i)}^T$
7: $\quad \widehat{m}_k^i \leftarrow \widehat{m}_k^{i-1} + 1$
8: $\quad \widehat{m}_j^i \leftarrow \widehat{m}_j^{i-1}, \forall j \in \Omega_c \setminus \{k\}$
9: $\quad \widehat{\mathbf{S}}_{\phi(i)}^* \leftarrow \mathbf{e}_k$
10: **end for**
11: **return** $\widehat{\mathbf{S}}^*$

Remark 3: In line 5, instead of random choices adopted by BGA, an optimal "k" is selected to maximize the immediate increase of network throughput. If multiple k's are available, everyone is an eligible choice.

Lemma 4: Let $|\Omega|$ denote the cardinality of set Ω. In OGA, if $|\Omega^i| > 1$, then $\Omega^{i+1} \subset \Omega^i$ and $|\Omega^i| = |\Omega^{i+1}| + 1$, where $1 \leq i < M$.

Lemma 5: For an i ($i \in \Omega_u$), in BGA and OGA, if $\widehat{m}_j^{i-1} = m_j^{i-1}$ ($\forall j \in \Omega_c$), then $\widehat{m}_j^t = m_j^t$ ($\forall j \in \Omega_c$), where $t = i + |\Omega^i| - 1$ ($t \in \Omega_u$).

Theorem 5: In OGA, $\widehat{\mathbf{S}}^*$ is a solution to (5).

Proof: Clearly, $\widehat{\mathbf{S}}^*$ is a pure-strategy NE. Let $\phi(i)$ be the input of both BGA and OGA. And let $i \leftarrow 1$ and $t \leftarrow i + |\Omega^i| - 1$. If $t \in \Omega_u$, since $\widehat{m}_j^{i-1} = m_j^{i-1} = 0, \forall j \in \Omega_c$, then by Lemma 5, $\widehat{m}_j^t = m_j^t, \forall j \in \Omega_c$. Now let $i \leftarrow t + 1, t \leftarrow i + |\Omega^i| - 1$. Again, if $i, t \in \Omega_u$, for the same reason ($\widehat{m}_j^{i-1} = m_j^{i-1}, \forall j \in \Omega_c$), we have $\widehat{m}_j^t = m_j^t, \forall j \in \Omega_c$ (Lemma 5). Iteratively, let $i \leftarrow t + 1, t \leftarrow i + |\Omega^i| - 1$ and apply Lemma 5 repeatedly until $i > M$ or $t > M$. Since both i and t keep increasing during the iteration, and that M is finite, this iterative process stops after finite steps. When the iteration stops ($i > M$ or $t > M$), $\widehat{m}_j^{i-1} = m_j^{i-1}, \forall j \in \Omega_c$. Thereby, $R(\widehat{\mathbf{S}}^{i-1}) = R(\mathbf{S}^{i-1})$, where $\mathbf{S}^{i-1} \triangleq [\mathbf{S}_{\phi(1)}^*, \ldots, \mathbf{S}_{\phi(i-1)}^*, 0, \ldots, 0]_{N \times M}$. It is sufficient to consider the following two cases:

Case 1, $i > M$: Since $i - 1 \in \Omega_u$, then $i = M + 1$ and $\mathbf{S}^{i-1} = \mathbf{S}^M = \mathbf{S}^*$, which means that all pure-strategy NE reach the same network throughput.

Case 2, $i \in \Omega_u, t > M$: To achieve a pure-strategy NE, the rest of players ($\phi(i)$ to $\phi(M)$) need to select $M - i + 1$ distinct channels from the channel set Ω^i. Since $R(\widehat{\mathbf{S}}^{i-1}) = R(\mathbf{S}^{i-1})$, achieving an optimal NE with maximum throughput is equivalent to maximize the increase of throughput in the next $M - i + 1$ loops. Clearly, in OGA line 5 guarantees this: in every loop line 5 picks out a channel from Ω^i with the maximum throughput increase, and after $M - i + 1$ loops, the $M - i + 1$ channels with the highest throughput increases are selected.

So far it is proved that $\widehat{\mathbf{S}}^*$ achieves the maximum network throughput for a fixed ϕ. Since the order (i.e., ϕ) of making sensing decisions does not affect the achieved throughput, the throughput of $\widehat{\mathbf{S}}^*$ is maximum for all ϕ's. Therefore, by Theorem 4, $\widehat{\mathbf{S}}^*$ is a solution to (5). ■

As a brief review, in this section, at first, BGA is proposed to calculate every pure-strategy NE. Afterwards, OGA is proposed as a modified version of BGA to solve (5). It should also be emphasized that both BGA and OGA are fast and costless with the time and space complexity of $\mathcal{O}(M \log N)$ and $\mathcal{O}(N)$, respectively.

V. Practical Protocol Design

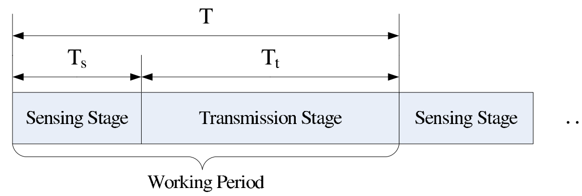

Fig. 1. Structure of working period

In this section, we describe a MAC protocol to implement OGA in a distributed way with negligible communication overhead. We assume there exists a designated common control channel (e.g., an ISM band) available to all SUs.

In our MAC protocol, the time axis is divided into a series of *working periods* with period length of T. Each working period consists of two stages, the *sensing stage* with interval length of T_s and the *transmission stage* with length of T_t, as dipicted in Fig. 1. The SUs are synchronized and can sense and transmit on one channel only in sensing and transmission stage, respectively. If an SU has a transmission task in t, it is forced to wait until the next sensing stage. We call an SU *transmitter* if it has a transmission task at the beginning of working period.

In sensing stage, each transmitter chooses one channel to sense and broadcasts its decision to other transmitters. Each transmitter maintains a *decision table* $[m_1, \ldots, m_N]$ where m_i records the number of transmitters that have decided to sense channel i. It follows the steps below to make the sensing decision.

First, the transmitter suspends its transmission tasks (if any) inherited from last transmission stage, clears out its decision table (i.e., set all $m_i = 0$), and tunes its transceiver to the control channel. Then it listens for a certain amount of time randomly distributed in $[0, T_b]$, where T_b is the maximum backoff time. If it does not hear a sensing decision from another transmitter, it will selfishly make its own decision. Since it does not know whether it is the last transmitter to make sensing decision, the best strategy for it is to maximize the immediate obtainable interest as indicated in OGA[4]. That is, first calculate $\Omega \leftarrow \{p \mid p = \arg\max_{j \in \Omega_c} U(j, m_j + 1)\}$, then select a k ($k \in \Omega$) with the maximum increase of network throughput[5] (line 5 of OGA). Afterwards, it broadcasts its sensing decision of k and tunes the transceiver on channel k to start sensing immediately.

If the transmitter receives a sensing decision of k from others during its listening interval, it updates its decision table by adding m_k by 1. Then it listens again for a random amount of time in $[0, T_b]$. If no sensing decision received, it calculates, broadcasts and senses in the same way as above. Otherwise, it updates the decision table, and listens again. If the remaining time in sensing stage is not enough to get an ideal sensing result, the transmitter has to wait until the next round of sensing stage.

When the sensing stage is over, it comes to the transmission stage. If the selected channel is sensed to be idle, transmitters will compete to access the channels following a standard CSMA approach. The transmission continues until the end of transmission stage. Then a new round of channel sensing will be initiated following the similar approaches mentioned above.

It can be seen that at the worst case (i.e., all SUs are transmitters), in sensing stage the above protocol will broadcast only M control packets in total to inform other SUs that new sensing decisions have been made. Every SU operates independently without any central controllers. In summary, OGA can be practically applied in a distributed way with negligible communication overhead.

VI. Conclusion

In this paper, we formulated a noncooperative game to study the optimal sensing policy with maximum network throughput in noncooperative CRN. We proposed a greedy algorithm, i.e., BGA, with a simple structure to find a pure-strategy NE in a fast and costless way. We proved that BGA is sufficient to calculate every pure-strategy NE. We then proposed OGA as a modified version of BGA to reach an optimal pure-strategy NE with maximum network throughput. A MAC protocol to implement OGA was also presented to show that it can be practically applied in a distributed way with negligible communication overhead.

[4]If it knows that it is not the last one to make sensing decision, it may violate the greedy behavior described in OGA by not maximizing the immediate achievable interest. For example, consider a 3-player 2-channel game where $D_1 = 12, D_2 = 8$ and $U(k, m_k) = \frac{1}{m_k} D_k$. If the first one knows that there are two other transmitters who have not yet made sensing decisions, it knows that the final decisions will be two to sense channel 1 and one to sense channel 2. Therefore, it will intentionally choose sepctrum 2 instead of 1 to obtain the final maximum utility of 8, which violates the behavior of selecting 1 in OGA.

[5]If there are multiple k's, anyone is qualified for selection.

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